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## ABSTRACT

Selecting programs, textbooks, and curriculum materials is one of the most important decisions educators make. An Expert Panel on Mathematics and Science was established by the U.S. Department of Education to develop a high-quality, research-based process for selecting programs and to use that selecting process to identify exemplary and promising programs. This booklet describes eight mathematics programs designated as exemplary or promising. A detailed explanation of the submission and selection process and an explanation of how the programs were separated into exemplary and promising categories is included. The four characteristics used for the selection criteria include quality of program, usefulness to others, educational significance, and evidence of effectiveness and success. Each characteristic has multiple criteria with specific indicators of achievement. Each program is described in terms of the four selection characteristics and professional development resources, plus program costs, product contact information, and the table of contents of each book in the program are provided. (ASK)

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U.S. DEPARTMENT OF EDUCATION'S MATHEMATICS AND SCIENCE

E X P E R T P A N E L

*Exemplary  
& Promising*  
**MATHEMATICS**  
P R O G R A M S

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# INTRODUCTION

While U.S. student achievement in mathematics has been increasing over the past two decades, international studies show that the achievement of our students lags behind that of students in most other advanced nations, particularly in middle and high school. Both research and common sense tell us that what students learn depends upon what they are taught. Moreover, what students are taught is largely determined by the programs, textbooks, and other curriculum materials schools choose. One cause for our students' relatively poor performance can be found in our typical mathematics textbooks, which are too often unfocused and undemanding. In our era of increasing technology and information, it is essential that all students have access to challenging and high-quality mathematics programs that show evidence of increasing student mastery of essential mathematical knowledge and skills.

Selecting programs, textbooks, and curriculum materials is thus one of the most important decisions educators make. The decision-making process represents a critical investment of time and resources. The stakes are high: our children's future and the ability of our citizens to manage and solve the complex questions of our age. For these reasons, an Expert Panel on Mathematics and Science has been established to develop a high-quality, research-based process for selecting programs and to use that selection process to identify exemplary and promising programs.

## THE EXPERT PANEL SYSTEM

In 1994, Congress directed the Office of Educational Research and Improvement (OERI) to establish "panels of appropriate qualified experts and practitioners" to evaluate educational programs and recommend to the Secretary of Education those programs that should be designated as exemplary or promising. The purpose of these panels is to provide teachers, administrators, policymakers, and parents with solid information on the quality and effectiveness of programs and materials so that they can make better-informed decisions in their efforts to improve the quality of student learning.

An Expert Panel on Mathematics and Science Education was established to develop and oversee a valid and reliable process for identifying and designating exemplary and promising programs. The Expert Panel is composed of educators, scientists, mathematicians, and policymakers with extensive experience in mathematics and science education. In the first year of operation, 1998, the Expert Panel reviewed mathematics programs.

## THE REVIEW PROCESS

Before developing criteria and procedures for reviewing programs, the Expert Panel for Mathematics and Science Education assessed the status of mathematics education. It found that 43 states have adopted or substantially incorporated the recommendations from the national standards documents into their own state standards and curriculum frameworks. It also found that educators across the country are now asking for curriculum materials and programs that translate the standards into a useful form for their classrooms. Therefore, in this round of review, the Expert Panel for Mathematics and Science Education sought out



programs that exemplify the high level and challenging mathematics called for in the National Council of Teachers of Mathematics (NCTM) standards and the American Association for the Advancement of Science (AAAS) benchmarks. In future reviews the emphasis may be different from this round of review. The criteria used in the review can be found on page 5.

The Expert Panel had an open and widely publicized submission process that encouraged applications from any program that might meet the review criteria. In this first year, 61 mathematics programs were voluntarily submitted for review. Almost 100 teachers and other researchers and practitioners with expertise in mathematics were trained for three days in the review process. Each program submitted to the Expert Panel process was reviewed by two field-based teams with two members each—a total of four individuals reviewing each submission. These teams reviewed the quality of the program, its usefulness to others, and its educational significance based on materials submitted by the developers. Programs that received high ratings from this procedure were then reviewed by program evaluation experts who assessed the quality of the evaluation data and the claims of effectiveness made by the submitters. The full Expert Panel then reviewed all the programs, along with the ratings and comments of the review teams, to determine which programs to recommend to the Secretary of Education as exemplary or promising.

Exemplary programs must be highly rated on quality, usefulness to others and educational significance and must provide *convincing* evidence of effectiveness in *multiple* sites with *multiple* populations. Promising programs must be rated high in terms of quality, usefulness for others to implement, and educational significance, in addition to providing *preliminary* evidence of effectiveness in one or more sites.

This publication provides descriptions of the five exemplary and five promising programs designated by the Secretary of Education in 1999. Contact information for each program is also provided. In the program summaries that follow, the sections “Program Descriptions and Curriculum Materials” and “Professional Development Resources and Program Costs” were prepared based on information provided by the developers. The remaining sections—“Program Quality,” “Usefulness to Others,” “Educational Significance,” and “Program Effectiveness and Success”—are based on the assessments of the expert reviewers.

## HOW TO USE THESE RECOMMENDATIONS

The purpose of the Expert Panel system is to help schools make better-informed decisions in selecting programs and materials. It provides both an example of a rigorous, analytical process for selecting curriculum and information about materials that were submitted to the process and designated as exemplary or promising. This is a tool to help schools as they make the important decisions on appropriate materials and programs for their mathematics program. The following are some examples of ways that the recommendations of the Expert Panel may be used:

- Local school or school district selection committees for mathematics programs and materials may use these recommendations to decide which materials they will review in greater depth;
- Central office staff may contact the developers or implementation sites to get more information on programs in order to provide more information to schools on options for mathematics programs;

- Regional or state entities could host conferences that highlight the selected programs and materials so that schools and communities would have a better sense of what the programs offer;
- Individual schools, school districts, or state boards of education could use the Expert Panel selection criteria and review process as a basis for analyzing materials and programs under consideration for selection;
- Individual schools, school districts, or state boards of education could use the Expert Panel selection criteria and review process as a point of comparison for updating their own curriculum selection process; and
- Teacher preparation programs can use these programs and materials in preservice or inservice education programs training teachers in mathematics or on standards.

# EVALUATION CRITERIA

The following criteria and indicators were used to evaluate the mathematics programs submitted to the Expert Panel.

## A. QUALITY OF PROGRAM

**Criterion 1.    *The program's learning goals are challenging, clear, and appropriate for the intended student population.***

- Indicator a. The program's learning goals are explicit and clearly stated.
- Indicator b. The program's learning goals are consistent with research on teaching and learning or with identified successful practices.
- Indicator c. The program's learning goals foster the development of skills, knowledge, and understandings.
- Indicator d. The program's learning goals include important concepts within the subject area.
- Indicator e. The program's learning goals can be met with appropriate hard work and persistence.

**Criterion 2.    *The program's content is aligned with its learning goals, and is accurate and appropriate for the intended student population.***

- Indicator a. The program's content is aligned with its learning goals.
- Indicator b. The program's content emphasizes depth of understanding, rather than breadth of coverage.
- Indicator c. The program's content reflects the nature of the field and the thinking that mathematicians use.
- Indicator d. The program's content makes connections within the subject area and between disciplines.
- Indicator e. The program's content is culturally and ethnically sensitive, free of bias, and reflects diverse participation and diverse student interests.

**Criterion 3.    *The program's instructional design is appropriate, engaging, and motivating for the intended student population.***

- Indicator a. The program's instructional design provides students with a relevant rationale for learning this material.
- Indicator b. The program's instructional design attends to students' prior knowledge and commonly held conceptions.
- Indicator c. The program's instructional design fosters the use and application of skills, knowledge, and understandings.
- Indicator d. The program's instructional design is engaging and promotes learning.
- Indicator e. The program's instructional design promotes student collaboration, discourse, and reflection.
- Indicator f. The program's instructional design promotes multiple and effective approaches to learning.
- Indicator g. The program's instructional design provides for diverse interests.



**Criterion 4.   *The program's system of assessment is appropriate and designed to inform student learning and to guide teachers' instructional decisions.***

- Indicator a. The program's system of assessment is an integral part of instruction.
- Indicator b. The program's system of assessment is consistent with the content, goals, and instructional design of the program.
- Indicator c. The program's system of assessment encourages multiple approaches and makes use of diverse forms and methods of assessment.
- Indicator d. The program's system of assessment probes students' abilities to demonstrate depth, flexibility, and application of learning.
- Indicator e. The program's system of assessment provides information on students' progress and learning needs.
- Indicator f. The program's system of assessment helps teachers select or modify activities to meet learning needs.

## **B. USEFULNESS TO OTHERS**

**Criterion 5.   *The program can be successfully implemented, adopted, or adapted in multiple educational settings.***

- Indicator a. The program provides clear instructions and sufficient training materials to ensure use by those not in the original program.
- Indicator b. The program is likely to be successfully transferred to other settings.
- Indicator c. The program specifies the conditions and resources needed for implementation.
- Indicator d. The program's costs (time and money) can be justified by the benefits.

## **C. EDUCATIONAL SIGNIFICANCE**

**Criterion 6.   *The program's learning goals reflect the vision promoted in national standards in mathematics education.***

- Indicator a. The program's learning goals and subject matter content are consistent with national standards.
- Indicator b. The program's pedagogy and assessment are aligned with national standards.
- Indicator c. The program promotes equity and equal access to knowledge, as reflected in national standards.

**Criterion 7.   *The program addresses important individual and societal needs.***

- Indicator a. The program is of sufficient scope and importance to make a significant difference in student learning.
- Indicator b. The program contributes to increases in teachers' knowledge of effective teaching and learning.

Indicator c. The program:

- is designed to improve learning for a wide spectrum of students OR
- serves to meet the special learning needs of under-served students OR
- serves to meet the special learning needs of students whose interests and talents go beyond core mathematics education.

## D. EVIDENCE OF EFFECTIVENESS AND SUCCESS

**Criterion 8.** *The program makes a measurable difference in student learning.*

**Exemplary Programs**, in addition to satisfying Criteria 1–7, must provide *convincing* evidence of effectiveness in *multiple sites with multiple populations* regarding *two or more* of the indicators below. The items must include either both indicators from Part I or one indicator from Part I and one indicator from Part II. Providing evidence of two indicators from Part II is not sufficient.

### Part I

- Indicator a. The program has evidence of gains in student understanding of mathematics.
- Indicator b. The program has evidence of gains in inquiry, reasoning, and problem solving skills.

### Part II

- Indicator c. The program has evidence of improvements in course enrollments, graduation rates, and post-secondary school attendance.
- Indicator d. The program has evidence of improvements in attitudes toward learning.
- Indicator e. The program has evidence of narrowing the gap in achievement or accomplishment between disaggregated groups.
- Indicator f. The program has other evidence of effectiveness or success.

**Promising Programs**, in addition to satisfying Criteria 1–7, must provide *preliminary* evidence of effectiveness in *one or more sites* for *at least one* of the indicators below:

- Indicator a. The program has evidence of gains in student understanding of mathematics.
- Indicator b. The program has evidence of gains in inquiry, reasoning, and problem solving skills.
- Indicator c. The program has evidence of improvements in course enrollments, graduation rates, and post-secondary school attendance.
- Indicator d. The program has evidence of improvements in attitudes toward learning.
- Indicator e. The program has evidence of narrowing the gap in achievement or accomplishment between disaggregated groups.
- Indicator f. The program has other evidence of effectiveness or success.



*Cognitive* ( EXEMPLARY PROGRAM  
*Tutor*<sup>TM</sup>  
ALGEBRA

# COGNITIVE TUTOR™ ALGEBRA

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

Carnegie Learning's *Cognitive Tutor™ Algebra* is a full-year, first-year algebra course that integrates technology in its instructional design. It can be used by students in Grades 7–12 or college undergraduates, and it addresses topics in a typical high school or middle school Algebra I course. The program has also been referred to as PACT Algebra or Pump Algebra.

Included in the program are the full-course curriculum (text, assignments, assessments, and activities), curriculum-integrated software (*Cognitive Tutors™*), user's guide to accompany the software, and teacher's guide. Research has shown that staff development and teacher and technical support are essential to successful implementation; this level of support is included with the program.

An important feature of the program is the use of a new approach to educational software. The intelligent computer tutor is based on research in cognitive psychology and artificial intelligence and provides each student with an individualized coach or tutor. *Cognitive Tutor™* also provides instant feedback and assistance to the student as needed. The intelligent computer tutors (*Cognitive Tutors™*) were developed by cognitive psychologists and computer scientists in Carnegie Mellon University's PACT Center within the Human Computer Interaction Institute of the School of Computer Science.

The program is designed to have students work on cooperative problem-solving activities three days a week in the classroom and on similar individual computer-based problems in a computer laboratory the other two days. Students investigate and solve real-world problem situations with increased attention placed on the entire problem-solving process. Students link numeric, verbal, graphic, and symbolic representations, while using tools such as spreadsheets and calculators.

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## Professional Development Resources and Program Costs

Carnegie Learning's *Cognitive Tutor™ Algebra I* program includes: a master loose-leaf textbook (370 pages), teacher guide, curriculum guide, student assignments, classroom assessment, and the *Cognitive Tutor™* software. The site license for this comprehensive algebra curriculum is \$25,000 per site. Quantity discounts are applied for multi-site districts.

Carnegie Learning's *SchoolCare™* Program includes: initial teacher preparation (a 4–5 day, centralized training course), ongoing updates

and upgrades to the software and the printed curriculum materials, and help-desk telephone and email weekday support. Site visits are conducted as required. The annual cost of *SchoolCare™* is \$2,500 per site. Discounts are applied for multiple sites. On-site teacher preparation can be scheduled for \$1,500 per day. Additional staff consulting services are quoted on a custom basis.

The *Cognitive Tutor™ Algebra* program is currently in use among ethnically and economically diverse populations nationwide in urban, suburban, and rural areas and among middle school, high school, and college undergraduate students. As of February, 1999, it is in use in over 75 schools. It has also been used by special needs students in self-contained and inclusive classrooms. Reviewers found the program particularly appropriate for the urban underachiever. Reviewers noted that the *Cognitive Tutor™ Algebra* program provides excellent implementation support for schools in the form of support materials and teacher inservice necessary for successful implementation of computer-assisted instruction.

## Educational Significance

*Cognitive Tutor™ Algebra*, which is intended for use in a typical high school or middle school Algebra I course, includes topics for a full year of study. It emphasizes communication, connections, reasoning, problem solving, and the use of technology as a tool. The program addresses the NCTM standards via multiple representations of problems, cooperative learning, technology, and problem solving. Students are provided the time to explore sound mathematics and ask questions that help them develop concepts.

The *Cognitive Tutor™* provides many positive aspects to learning: students are able to receive instant feedback customized to their individual approach; weak skill areas are assessed and addressed on an individual student basis; it is an active, hands-on learning tool; and the problem situations apply to everyday activities. Additionally, it allows for the option of a self-paced, individualized learning environment that can serve a wide spectrum of individual needs. Teachers have access to assessment reports that enable them to provide additional instruction targeted at each student's identified needs.

## PROGRAM QUALITY

The mathematical content of the *Cognitive Tutor™ Algebra* program is sound and follows the NCTM standards, for instance, by asking students to use variables and functions to model mathematical situations. It also integrates other major strands of mathematics, such as statistics and geometry, with algebra and focuses on depth rather than breadth of coverage. The program is built around active learning as evidenced by the activity-oriented student text. Many of the problem-solving activities use real-world situations. These problem-solving activities require that students use written and oral communication and technology. The program provides familiarity and practice with problem-solving methods, algebraic notation, algorithms and geometric representations.

Reviewers found the program's instructional design motivating and engaging because it integrates a variety of approaches, such as use of a graphing calculator and *Cognitive Tutor™*. Additionally, meaningful, real-life applications through problem solving make this program relevant to the student. Interaction among students is motivating and keeps student interest level high. The program is designed to accommodate various learning styles and employs several different teaching and learning strategies in an effort to accomplish its goals.

The ongoing, varied approach to assessment is consistent with the program design. It also includes a primary computer component. It intends to help students improve in areas of weakness and lets them know their strengths. The wide variety of assessment methods addresses different learning styles.

## PROGRAM EFFECTIVENESS AND SUCCESS

*Cognitive Tutor™ Algebra* has been designated an **Exemplary** mathematics program. As part of their evaluation, *Cognitive Tutor™ Algebra* developed assessments to measure both basic functional skills and higher-order reasoning and problem-solving skills recommended in the NCTM standards.

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## FOR PRODUCT INFORMATION CONTACT:

**Carnegie Learning, Inc.**  
372 N. Craig Street, Suite 101  
Pittsburgh, PA 15213

**Telephone:** (412) 683-MATH  
**Fax:** (412) 683-0544

**E-mail:** [info@carnegielearning.com](mailto:info@carnegielearning.com)

**WWW Home Page:** <http://www.carnegielearning.com/>

The program's assessments include a problem situation test that examines students' ability in applying an algebraic analysis to authentic problem situations and a representation test that examines students' success in translating among symbolic, graphical, tabular, and verbal representations. The standardized assessments include questions excerpted from the Iowa Algebra Readiness Test and a subset of Math SAT questions.

In year-end assessment results from three years (1993–1994 at three Pittsburgh high schools, 1994–1995 at three Pittsburgh high schools, and 1996–1997 at one Milwaukee high school) students participating in the *Cognitive Tutor™ Algebra* course scored 50–100% higher, which was statistically significant, on two project-developed, problem-solving tests than students in traditional Algebra I courses (the comparison group). Students in the program also scored higher, to a statistically significant degree, on the standardized assessments (SAT and Iowa) across the three studies.

Looking at data from one high school in Pittsburgh that adopted *Cognitive Tutor™ Algebra* in 1992-93, Project students were more than twice as likely as traditional Algebra I students to enroll in traditional Algebra II two years later, following a year of traditional geometry. At this school, 66% of students in *Cognitive Tutor™ Algebra* passed the Algebra I course as compared to 44% of traditional Algebra I students.

In a 1994–1995 study, college students using *Cognitive Tutor™ Algebra* scored significantly higher than control group students on performance-based assessments focusing on areas such as defining variables, making a table, writing equations, constructing a graph, finding slopes/intercepts, and finding points of intersection. Students using *Cognitive Tutor™ Algebra* scored at approximately the same level as the control group on final exams targeting algebraic manipulation. Using a survey to collect quantitative data on students' attitudinal change, the developer found that *Cognitive Tutor™ Algebra* students had less computer anxiety, to a statistically significant degree, than comparison students.

## FOR RESEARCH AND EVALUATION INFORMATION CONTACT:

**Pittsburgh Area Cognitive Tutor (PACT) Center**  
Human-Computer Interaction Institute, School of Computer Science  
Carnegie Mellon University  
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**Fax:** (412) 268-1266

**E-mail:** ken.koedinger@cs.cmu.edu

**WWW Home Page:** <http://act.psy.cmu.edu/ACT/awpt/>



Student Letter; Syllabus; Facts in Five; Get It Together  
 Cube Problem; Number Patterns  
 Telephone-Checkerboard  
 \$4/hour & Consultant Problems  
 U.S. Shirts  
 Proportional Reasoning 1-3  
 Percentages 1-3  
 Distributive Properties 1-3  
 Absolute Value and Inequalities  
 Areas 1-3  
 Highway, Sandpile, & Car Rental #1  
 Car Rental 2 & 3; Solving Problems 1  
 Solving Problems 2 & 3; Algebraic Equations and Their Graphs 1  
 Algebraic Equations and Their Graphs 2; Equations-Rates-Intercepts  
 Graphs and Linear Equations  
 Situations-Graphs  
 Graphing Lines  
 First Semester Assessment  
 Finding the Equation  
 Finding the Formula  
 Connections  
 Data Analysis  
 Systems  
 Squares  
 Vertical Motion  
 Quadratic Formula  
 Powers  
 Slope-Intercept  
 Linear Inequalities  
 General Form of a Linear Equation  
 Review and Practice Performance Assessments

COGNITIVE TUTOR™ ALGEBRA:

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*College*  
*Preparatory*  
**MATHEMATICS**  
( C P M )

EXEMPLARY PROGRAM

# COLLEGE PREPARATORY MATHEMATICS (CPM)

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

*College Preparatory Mathematics* (CPM) is a four-year secondary-school curriculum that integrates algebra and geometry content with conceptual understanding and problem-solving skills. The courses concentrate on six or seven core ideas which are introduced, practiced, revisited, and mastered throughout the year and are reinforced in subsequent courses. Problem solving and other skills are taught throughout. Mathematics 1 focuses on topics such as writing, solving, and graphing equations as well as symbol manipulation. Mathematics 2 focuses on geometric properties, spatial visualization, and conjecture and explanation. Mathematics 3 focuses on representation and modeling, intersections and systems, algorithms, and reasoning and communication. Mathematics 4 focuses on concepts of calculus, analysis of models, trigonometry, and advanced functions. The curriculum parallels traditional college preparatory course content (Algebra I, Geometry, Algebra II, and Mathematical Analysis/Pre-Calculus).

Each course includes a teacher's edition and student texts. The teacher's edition includes recommendations on how to teach each unit, resource pages, collections of test questions, suggestions for alternative assessments, a comprehensive parent guide, and information for parents and administrators to assist them with community communication. Student editions of Mathematics 1, Mathematics 2, and Mathematics 3 are available in Spanish. (For Mathematics 3, only the blackline masters are available in Spanish; the bound text is not.)

## PROGRAM QUALITY

Reviewers noted that *College Preparatory Mathematics*' learning goals are aligned with NCTM standards, and content strands are carried through all levels of the curriculum. The program is rigorous and provides familiarity and practice with numerous mathematical concepts (e.g., algebraic notation, algorithms, and geometric representations in Mathematics 1). The reviewers found that the overall program goals are well aligned, challenging, clear, and appropriate for the intended student population (lower level ability to advanced students). Each of the four courses is built on a few core ideas that are developed and deepened over a four-year period, thereby allowing students more time to master a concept.

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## Professional Development Resources and Program Costs

Teachers using the materials for the first time are required to attend 3 to 5 days of introductory workshops during the summer and 5 all-day workshops during the school year. Workshops, offered by the developers of CPM, are held in more than 15 locations throughout California and at several sites around the United States. The workshops include introductions to the CPM methodologies and course content, as well as alternative assessment, cooperative learning, questioning strategies, and classroom management skills. The workshop series ranges between \$375–\$750 per teacher (depending on the level of service from CPM and the location of the workshop site), plus the cost of up to 5 days released time. A free bi-monthly newsletter and a Web site

also provide teacher support. Based on previous experience of the developers, CPM materials are implemented most effectively when new CPM teachers work with a partner or group of colleagues in a school or district.

Student texts cost, per student, approximately \$21 for a 2-volume soft-bound set. Hardbound editions of Algebra and Geometry are \$38. Teachers' editions cost \$45 per course, and supplies cost approximately \$100 per year. Additionally, scientific calculators are recommended for Mathematics 1 and 2, and graphing calculators are recommended for Mathematics 3 and 4.

The program strategies emphasize active learning and group work; students are introduced to problem solving, communication, and reasoning through laboratory experiments and real-world applications. Support materials are provided in the student texts to help them review and evaluate their progress. The reviewers noted ample evidence for the application of skills through problems that engage the students in both individual and collaborative work and address a variety of learning styles. Concepts are developed through guided instruction, individual and team work, tactile and kinesthetic activities, data collection, classwork, and homework. Students are encouraged to develop a positive attitude and become more aware of their own thinking about problems and to describe their efforts both orally and in writing.

An assessment handbook is provided in the teacher editions and outlines a variety of options for integrating assessment into instruction, e.g., investigations, portfolios, projects, presentations, problem solving, and daily performance assessment. The wide variety of approaches presented in the teacher's program materials include methods for assessing depth, flexibility, and application of learning. The student self-assessment component was viewed by the reviewers as a strong component of the assessment handbook.

## PROGRAM EFFECTIVENESS AND SUCCESS

*College Preparatory Mathematics* has been designated an **Exemplary** mathematics program. CPM has consistent evidence of improved student performance from a variety of studies that employ comparison groups and large sample sizes. Nine separate studies examined the achievement of approximately 30,000 California students in diverse settings. A variety of instruments was used to assess growth in mathematics achievement, including multiple choice assessments from the Math Diagnostic Testing Program (a well-established program that produces multiple-choice examinations for use by California high schools to provide diagnostic data and by colleges as placement tests), open-ended written response items which were processed by the University of California at Davis, the SAT mathematics exams, and California's Golden State Examination (a test to assess students in many disciplines for high achievement).

In three studies that asked students to provide written responses to open-ended questions assessing their inquiry, reasoning, and problem-solving skills, evidence consistently favored CPM students of both genders and all ethnicities over non-CPM students. The 1992 and 1993

## Usefulness to Others

Reviewers noted that the program is well developed with a solid curriculum and supportive teacher training component. The program has been used in both accelerated and regular classes from Grade 7 to early college, in block scheduling structures, and in a variety of geographic and multi-ethnic locations that include non-native English speakers, and students with learning disabilities. CPM courses are offered in more than 600 schools and have been used by more than 2,000 teachers and 1,500,000 students.

Reviewers noted that due to the cost and time requirements of the teacher workshops, CPM would be most easily implemented in districts with ample professional development resources.

## Educational Significance

The CPM program is consistent with the NCTM standards that suggest that real-life problems be used to show students that the mathematical concepts they are learning will be used after they leave the classroom. Each unit in CPM is based on real-life themes and built around appropriate mathematical concepts.

*continued on next page...*

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## FOR FURTHER INFORMATION CONTACT:

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**Telephone:** (888) 808-4CPM or (916) 391-3301  
**Fax:** (916) 444-5263

**E-mail:** hoey@cpm.org

**WWW Home Page:** <http://www.cpm.org/>

studies indicated that CPM students outscored their non-CPM counterparts to a statistically significant degree on both the algebra and geometry examinations. The 1994 study found that CPM students outscored the non-CPM comparison group to a statistically significant degree on the algebra examination at one of the two test sites.

Comparisons of SAT averages in four California schools before and after introduction of CPM, generally between 1994 and 1996, show increases in average school SAT scores of 13, 23, 30, and 39 points. These represent increases in their rankings among schools by 8 to 13 percentile points. No attempt was made to compute statistical significance.

A study at one junior high school comparing the first four years of CPM (1993–1996) with the preceding four years of non-CPM classes (1989–1992) showed that CPM produced statistically significant increases in the number of students qualifying for the state honors program based on performance on the California Golden State Examinations in algebra and geometry. Comparing any single non-CPM year with any CPM year, CPM students did better to a statistically significant degree, except for the algebra comparison of 1990 with 1993.

A study of two high schools comparing CPM students in honors algebra who later graduated in 1996 to non-CPM students in honors algebra and geometry classes who later graduated in 1994, 1995, or 1996 showed a greater percentage of CPM graduates taking calculus four years later. This difference was statistically significant. At a third high school, no calculus classes were offered until four years after CPM had been introduced. Now 22 out of 24 of those students (92%) take calculus.

Several studies examined possible differences in achievement produced by CPM for both genders, students of various ethnic groups, and students at different places on the performance spectrum. Results demonstrated that CPM works equally well for students of all characteristics.

### **Mathematics 1: (Algebra 1)**

Getting Started: Working in Teams  
Difference of Squares: Organizing Data  
The Kitchen Floor: Area and Subproblems  
The Burning Candle: Patterns and Graphs  
Choosing a Phone Plan:  
    Writing and Solving Equations  
Estimating Fish Populations:  
    Numerical, Geometric, & Algebraic Ratios  
World Records:  
    Graphing and Systems of Linear Equations  
The Big Race: Slopes and Rates of Change  
The Amusement Park: Factoring Quadratics  
The Birthday Party Piñata:  
    Writing Equations from Diagrams  
Yearbook Sales:  
    Exponents and Quadratic Equations  
The Cola Machine: Functions and Equality  
Problem Solving and Inequality  
The Rocket Show:  
    More About Quadratic Equations

### **Mathematics 2: (Geometry)**

Prelude  
Riding a Roller Coaster:  
    Perimeter, Area, Graphing, & Equations  
Convincing Your Team: Beginning Proofs  
Problem Solving and Geometry Fundamentals  
The TransAmerica Pyramid: Spatial Visualization  
Congruence and Triangles  
Tool Kit: Writing Proofs  
The Height of Red Hill: Trigonometry  
The Trekee's Clubhouse Logo: Similarity  
Urban Sprawl: Polygons, Area, and Proof  
The One-Eyed Jack Gold Mine: Circles and Solids  
The Poison Weed: Geometric Probability  
Going Camping: 3D and Circles

### **Mathematics 3: (Algebra 2)**

Shrinking Arrows: Exploring Functions  
The Bouncing Ball: Sequences  
Fast Cars and Depreciation: Exponential Functions  
The Gateway Arch:  
    Parabolas and Other Parent Graphs  
The Toy Factory: Linear Systems and Matrices  
The Case of the Cooling Corpse: Logarithms and  
    Other Inverses  
At the County Fair:  
    Polynomials and General Systems  
The Circle of Terror: Circular Functions  
The Search for Sunken Treasure:  
    More Triangles and Trigonometry  
Probability and Policy Making  
The Ice Cream Shoppe: Counting and Sampling  
Targets and Triangles: Elementary Statistics

### **Mathematics 4:**

#### **(Mathematical Analysis/Pre-Calculus)**

The Big Game and Radioactivity:  
    Introduction to Models  
Rocket Launch: Area Between Curves  
Introduction to Logarithms  
The Spring Problem: Sinusoidal Functions  
Algebra for College Math Courses:  
    Big Ideas and Little Tricks  
The Next Wave: More Modeling and Trigonometry  
Running on Empty:  
    Modeling and Statistical Analysis  
The New Pilot: Vectors and More Trigonometry  
To Infinity and Beyond: Limits  
Introduction to Polar Coordinates  
Space Telescope: Rates of Change  
Conic Sections:  
    Readiness for College Mathematics Texts  
Take Me Out to the Ball Game:  
    Parametric Equations  
Linear Transformations: Applications of Matrices  
Where's the Money? Series  
Appendix A: Quality Control and SAT Scores:  
    One Variable Statistics  
Appendix B: Yearbook Sales: Matrices  
Skill Builders (Extra Practice)  
Extra Problems

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*Connected*

EXEMPLARY PROGRAM

# MATHEMATICS

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# CONNECTED MATHEMATICS

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

*Connected Mathematics*, developed by the Connected Mathematics Project (CMP), is a comprehensive, problem-centered mathematics curriculum designed for all students in Grades 6–8. Each grade level curriculum is a full-year program, and in each of the three grade levels, topics of number, algebra, geometry/measurement, probability and statistics are covered in an increasingly sophisticated manner. The program seeks to make connections within mathematics, between mathematics and other subject areas, and to the real world.

The curriculum is divided into units, each of which contains 4 to 7 investigations. Each investigation contains 1 to 5 major problems that the teacher and students explore in class. Extensive problem sets, called *Applications*, *Connections*, and *Extensions*, are included for each investigation to help students practice, apply, connect, and extend these understandings. Each investigation culminates in a *Mathematical Reflections* activity.

The teacher's edition of each unit provides explicit instructions for the teacher explaining what should be done lesson by lesson to improve student understanding of the material covered. Materials also discuss the mathematics underlying the investigations, student assessment, and instructional roles of the teacher in orchestrating the investigations and the classroom. Teachers are provided with appropriate questions or activities to use, suggestions of what to do or look for, and an assessment package. Materials, time-lines, appropriate homework assignments, and essential vocabulary are all included with the program.

## PROGRAM QUALITY

The comprehensive, explicit goals of the *Connected Mathematics* program are well organized and address NCTM standards. Reviewers found that the broad array of student activities aligned with each goal provides students with appropriate and engaging opportunities for learning. Both individual and group activities require the student to develop mathematical concepts and algorithms. Each unit clearly strives to develop both student and teacher knowledge of mathematics that is rich in connections and deep in understanding and skill. The lessons are age-appropriate, and the sequencing of problem tasks within a unit, within a strand, and within a grade level helps students progress toward a strong understanding of the five targeted areas of mathematics.

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## Professional Development Resources and Program Costs

Upon adoption of the program, the publisher's regional consultants will provide a comprehensive program orientation. There is no charge for the initial in-service and program orientation. Additional professional development is available from the consultants for a fee. It is also recommended that a school adopting *Connected Mathematics* set up a program bringing teachers together at regular intervals to discuss issues and share knowledge during the first few years of implementation.

Materials for each grade consist of one student and one teacher book for each of the eight units, plus appropriate manipulatives that can be purchased separately. The catalogue prices are \$5.95 for one student book and \$16.50 for one teacher book. Teacher's Guide Packages,

which include teacher books for all 8 units of one grade and "Getting to Know Connected Mathematics," cost \$133 each. Teacher Resource Kits range from \$115–\$135 each. Discounts are available. A graphing calculator is recommended for the seventh and eighth grade classes. Other manipulatives needed for the units are common classroom items like square tiles, cubes and rulers. These can be purchased individually or in Student Materials Kits, which range from \$176–\$258. There is also the expectation that computer software will be provided by the schools that students can use in tandem with the curriculum. Five of the software applications may be downloaded at no charge from the Connected Mathematics Website.

*Connected Mathematics* has been used by students in a wide range of districts across the country, including geographically, economically, and ethnically diverse populations in rural, suburban, and urban areas.

Reviewers found the print support materials to be of high quality. The teacher's edition guides teachers through the problems and investigations and also lists all necessary materials for each lesson. Although the program depends upon the teacher's ability to understand, implement and persevere with this new program, the information for teachers in the lessons offers powerful assistance.

## Educational Significance

*Connected Mathematics* is a problem-centered curriculum that encourages individual, pair, and group work. It is replete with interesting, creative, and mathematically rich investigations and suggestions for their study. Problem solving, reasoning, communication, and connections are kept in the forefront of each activity and student learning is assisted by diagrams and pictures. Reviewers found that these methods address a wide array of ethnic and racially diverse populations.

The instructional and assessment strategies used are powerful and help a teacher to diagnose where students are struggling with the material. Teachers are supported by well-developed resources that help them teach the lesson. The program should considerably increase teachers' knowledge of effective teaching and learning.

Reviewers concluded that the instructional design of the program is well-conceived and potentially engaging and motivating for students. The program guides students through the lesson development and provides them with hands-on tasks and opportunities to design solutions to the tasks or projects. The materials appear to be challenging for the designated grade levels and are offered in a context relevant to students that takes into account their prior learning experiences and conceptions. The program promotes learning and student collaboration, discourse, and reflection. It provides multiple approaches to learning that include the use of calculators and computer activities. *Connected Mathematics* provides opportunities that challenge students of varied interests and abilities to achieve higher levels of understanding and skill.

Student assessment involves the use of varied and diverse measures to determine how well students are doing and is directly aligned with the content, goals, and design of the program. Some of these measures include standard problem-solving tests, partner quizzes, unit tests, projects, notebooks, self-assessments/student reflection and check-ups. Rubrics are also provided in the teacher's edition in order to evaluate student work. Emphasis is placed on engaging students in diverse activities that utilize manipulatives, the calculator and other tools to help students develop a conceptual and working knowledge of the key goals identified.

## PROGRAM EFFECTIVENESS AND SUCCESS

*Connected Mathematics* (CMP) is designated an **Exemplary** mathematics program. Strong evidence was found for the impact of the program on gains in students' understanding of mathematics, mathematical reasoning, and problem solving. The strength of the evidence was based on use of multiple measures, including tests aligned with the NCTM standards, and several types of comparisons.

In one study of approximately 2,500 students during the developmental stages of the program, sixth and seventh grade *Connected Mathematics* students, tested during the 1994–1995 school year, and eighth grade *Connected Mathematics* students, tested during the 1995–1996 school year, had significantly greater gain scores on several forms of the Balanced Assessment Project than did comparison students.

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(The Balanced Assessment Project was jointly designed by several major universities with support from the National Science Foundation to measure a variety of mathematical curriculum topics in an open response format with emphasis on communication, connections, reasoning, and problem solving.) The Iowa Test of Basic Skills was also given to the same set of students in 1994–1995 (sixth and seventh grade) and 1995–1996 (eighth grade). For the sixth and seventh grades there were no statistically significant differences between the CMP and comparison students, while in the eighth grade there were statistically significant differences in favor of the CMP classes.

In a study by other researchers of the effects of *Connected Mathematics* on students' understanding of proportional reasoning, seventh grade *Connected Mathematics* students, tested in the spring of 1995, scored 46% correct while students in the comparison class scored 23% correct. Eighth grade *Connected Mathematics* students, tested in the spring of 1996, scored 58% correct while students in comparison classes scored 28% correct. Tests for statistical significance were not conducted.

Since the publication of the materials, CMP continues to collect reports from school districts around the country that are using the project. Evidence from 1991–1996 in Michigan shows that the percentages of seventh grade students in one field site achieving satisfactory scores on the Michigan Educational Assessment Program increased by nearly 70% over the course of five years while the state increased by 61%. At the same time, the number of students achieving the low category decreased by 70% for the field site, while the state decreased 41% in this category.

Of 17 Texas schools using *Connected Mathematics*, 13 reported increases in the percentage of sixth grade students meeting the minimum expectations on the Texas Assessment of Academic Skills (TAAS). The increase in the 13 schools' scores ranged from 1 to 13 percentage points, an increase of 1.3% to 22% in the schools' scores from one year to the next, after one year of implementation. The scores for the state of Texas increased 4 percentage points, a 5.2% increase, from 1996 to 1997. After two years of implementation, from 1996–1998, all 17 schools showed increases in their TAAS scores ranging from 1 to 16 percentage points, an increase of 1.2% to 27.1 % in the schools' scores from one year to the next, while the state average increased 9 percentage points, an 11.7% increase. Tests for statistical significance were not conducted.

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In a series of Minnesota comparisons, in 1997 students in full-implementation *Connected Mathematics* schools (i.e., schools where the majority of students were in math classes taught by a *Connected Mathematics* teacher, and where the majority of *Connected Mathematics* teachers received training, orientation, and on-going support and follow-up provided by a *Connected Mathematics* mentor and colleagues) significantly outscored others in five of six comparisons on the State Basic Standards Mathematics Tests taken by eighth grade students and the CAT/5 taken by seventh grade students. In partial implementation sites in Minnesota, modest positive results were reported but not to the level of statistical significance.

## TO ORDER CONTACT:

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Number Theory—factors, multiples, primes, and composites

#### ***Data About Us***

Data investigation—formulating questions, gathering data, organizing and representing data, and interpreting data

#### ***Shapes and Designs***

Reasoning about 2-dimensional shapes and relationships among length, angle measure, and how shapes fit together

#### ***Bits and Pieces, Part I***

Understanding rational numbers—fractions, decimals, and percents; equivalence and order

#### ***Covering and Surrounding***

Reasoning about 2-dimensional shapes and measurement; area and perimeter

#### ***How Likely Is It?***

Reasoning under uncertainty, experimental and theoretical probabilities, equally-likely and unequally-likely events

#### ***Bits and Pieces, Part II***

Using rational numbers to solve problems, meaning of and skill in addition, subtraction, and multiplication of fractions and decimals

#### ***Ruins of Montarek***

Spatial visualization and spatial reasoning; architectural and isometric forms of representation

### 7th Grade

#### ***Variables and Patterns***

Understanding variables and representations of relationships, such as tables, graphs, and simple symbolic forms

#### ***Stretching and Shrinking***

Similarity in 2-dimensional shapes with congruence as a special case of similarity

#### ***Comparing and Scaling***

Proportional reasoning; ratio, proportion, rate and percent

#### ***Accentuate and Negative***

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#### ***Moving Straight Ahead***

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#### ***Filling and Wrapping***

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Probability; understanding and using expected value

#### ***Data Around Us***

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#### ***Looking for Pythagoras***

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#### ***Painted Cubes***

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Investigating equivalent expressions; solving linear and some quadratic equations

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Transformational geometry; connecting geometry and algebra

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*Core-Plus*

EXEMPLARY PROGRAM

MATHEMATICS

*Project*

# CORE-PLUS MATHEMATICS PROJECT

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

*Contemporary Mathematics in Context: A Unified Approach* is an integrated mathematical sciences curriculum for high schools, developed by the Core-Plus Mathematics Project (*Core-Plus*) to prepare students for success in college, in careers, and in daily life. The curriculum consists of a sequence of three core courses for all students, plus a fourth-year course, continuing the preparation of students for college mathematics. This fourth-year course is currently being field tested nationally. The program developers recommend that districts implementing the *Core-Plus* materials start with Course 1 and add the other courses one year at a time.

Each year of the curriculum features four interwoven strands: algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. The curriculum emphasizes mathematical modeling and features full use of graphing calculators. The curriculum is designed so that core topics are accessible to students of different interests and abilities. Differences in student performance and interest are accommodated by the level of abstraction to which topics are pursued and by the nature and degree of difficulty of applications. Comprehensive, curriculum-embedded assessment and supplementary assessment tasks allow monitoring of each student's performance in terms of mathematical processes, content, and dispositions.

## PROGRAM QUALITY

Reviewers found the learning goals of *Core-Plus* to be explicit and clearly stated. The goals lay out the important mathematical ideas that are part of each course as well as the mathematical strands that are threaded throughout each course. The variety of problems in each lesson involves work that should result in the achievement of the objectives of each unit, and the material is well-chosen to give all students a rationale for learning the mathematics and the motivation to learn through problems that interest them. Each unit clearly leads to the next unit, and problems and ideas take into account prior learning. Well prepared students will flourish in the program, but, more importantly, reviewers noted that weakly prepared students should also be successful.

The program draws connections between mathematics and everyday, real-life situations by developing essential mathematical concepts in the context of modeling real-world problems. A variety of

*continued on next page...*

## Professional Development Resources and Program Costs

Since teachers are encouraged to use different teaching methods, and because discrete mathematics and statistics topics are often new to many teachers, developers recommend that at least one lead teacher from a district attend a regional or national Core-Plus implementation workshop. Teacher workshops are available at the development site in Michigan as well as at regional locations. Costs range between \$300 and \$500 per person plus transportation, lodging and meals. The workshops allow for in-depth, hands-on experience with the materials, and also provide an opportunity to talk with other teachers who have experience with the program. Customized on-site workshops can be arranged through the publisher.

Each course consists of a two-volume set of student texts costing \$21 per volume. The complete package of teachers' materials is \$170 per course (both volumes). In addition, students need access to graphing calculators in each course. Software for the calculators, including a software guide for each course, costs \$38. Programs may be downloaded to all student calculators; the first download is from a computer that uses a linking connector.

approaches to learning such as whole-class discussion, small-group collaborative investigation, summarization, direct instruction, and individual application of learned concepts and skills are also incorporated into the program.

The assessment for this program is comprehensive, detailed and varied. The assessment package is integrated into the program's design and provides appropriate explanations for teachers to become comfortable with how to best use the available assessment opportunities. There is also a full collection of more traditional and alternative assessment instruments that accurately reflect the material lesson by lesson and unit by unit.

## PROGRAM EFFECTIVENESS AND SUCCESS

*Core-Plus* has been designated an **Exemplary** mathematics program. The program provided strong evidence of producing gains in students' understanding of mathematics and development of inquiry, reasoning and problem-solving skills. The strength of the evaluation of *Core-Plus* is based on large samples of students in a wide variety of geographic locations and evidence that applies to students across the range of achievement levels. Evaluation studies have included comparisons of performance with comparable control students in traditional mathematics classes as well as comparisons to national norms.

In a study that employed the Ability to Do Quantitative Thinking test, the mathematical reasoning subtest of the Iowa Tests of Educational Development, overall, the *Core-Plus* students outperformed students in the national norm group after one year of instruction. These performance levels were maintained as students progressed through two additional years of *Core-Plus* instruction. *Core-Plus* students' mean standard scores consistently increased by about 10 points per year, 4 to 6 points greater than norm groups with similar pretest means. Overall, *Core-Plus* students also outperformed, to a statistically significant degree, comparable control students who were tested at the end of one year. At the end of two years, *Core-Plus* students overall demonstrated greater growth than that of the comparison students, and these gains were found to be statistically significant in one of the five test schools.

On items taken from the 1990 or 1992 NAEP assessment of twelfth-grade students, *Core-Plus* Course 3 students scored higher than the NAEP samples in all content and

## *Educational Significance*

*Core-Plus* integrates many instructional strategies into its design, assuring that a diverse student population has the opportunity to learn mathematical concepts thoroughly. *Core-Plus* takes the recommendations of the NCTM standards and designs a program to achieve those standards. Reviewers noted that a program of this type, apart from how well it prepares students for work in college, is tremendously important for developing a mathematically literate society. It provides a vehicle for learning in real life contexts with strategies that meet a variety of student needs, and offers a rationale for the importance of understanding algebraic and geometric concepts in the world. The instructional strategies that are used in delivering this program such as experimentation, collaboration, use of technology, and project-based learning provide for development of other skills and mathematical habits of mind that students need as they work in society.

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process categories, particularly so on items in the content category of statistics and probability and on items in the process category of conceptual understanding. The NAEP-based test was administered in May 1997 to 1,292 students from 23 *Core-Plus* field-test schools.

Several comparisons of SAT and ACT college entrance exam scores of *Core-Plus* students and students in traditional curricula with similar pre-high school achievement were conducted in 1996–1997. *Core-Plus* students performed as well as, or better than (but not to a statistically significant degree), students in traditional curricula who had similar prior school achievement.

On project-developed, open-ended achievement post-tests of algebraic and geometric content, *Core-Plus* students taking Courses 1 and 2 scored higher on conceptual, application, and problem-solving tasks than comparison students in more traditional mathematics classes, overall resulting in large effect sizes ranging from 0.59–1.27 for two of the three subtests administered for each course. Interviews were conducted with students in both groups to understand more about the reasons for differences in approaches to problem solving. Overall, *Core-Plus* students appeared to have a more solid understanding than students in traditional classes of the connection between a context and problem situation and its mathematical model and of the connections among the table, graph, equation, and verbal representations of that model.

On a pre-post student belief survey about mathematics, *Core-Plus* Course 1 students demonstrated a statistically significant level of growth in confidence about their ability to solve mathematical problems and to reason mathematically, as compared to students in traditional mathematics courses. In addition, statistically significantly higher growth was evidenced in their interest in mathematics and their desire to take additional mathematics courses. Overall, they reported more positive attitudes about their mathematics course than did the comparison students. Specifically, *Core-Plus* students were more positive, to a statistically significant degree, than the comparison students about the realism, interest, and challenge of the investigations and problems in their mathematics courses and also about communicating, learning, and problem solving in cooperative groups and perceiving group problem solving as an important aid to their learning of mathematics.

There is some indication, from preliminary analysis, of positive effects of *Core-Plus* on females' achievement in mathematics. Analysis of the project-developed Course 1 post-test (e.g., algebraic concepts applied in realistic settings and linear equations) demonstrated that females in *Core-Plus* scored higher to a statistically significant degree than females in comparison classes on the Algebraic Concepts II subtest. The differences in scores between *Core-Plus* and comparison students were higher for the *Core-Plus* females than the *Core-Plus* males.

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*Interactive*

EXEMPLARY PROGRAM

# MATHEMATICS

PROGRAM (IMP)

# THE INTERACTIVE MATHEMATICS PROGRAM (IMP)

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

The *Interactive Mathematics Program* (IMP) is a four-year secondary, comprehensive, problem-based mathematics curriculum that integrates traditional content, such as algebra, geometry, and trigonometry, with other topics such as statistics and probability. The program prepares students to use mathematics and problem-solving skills in further education and on the job.

The four-year curriculum is designed to help all students develop a deep understanding of mathematical concepts and how to apply them. It challenges students to explore open-ended situations actively, in a way that resembles the inquiry method used by mathematicians and scientists in their work. Students routinely investigate specific cases, look for and articulate patterns, and make, test, and prove conjectures. Each year of the program consists of five units. Units of the IMP curriculum begin with a central problem or theme, which students explore and/or solve over the course of the unit. Solving a particular unit problem often requires concepts from several branches of mathematics, allowing students to see how a variety of ideas relate to each other. Each unit also includes supplemental problems providing flexibility to meet individual student needs. Years 1–3 of the *Interactive Mathematics Program* are currently available. Year 4 will be available for Fall 1999.

## PROGRAM QUALITY

Reviewers found *Interactive Mathematics Program's* content to be strongly aligned with its goals, and there was evidence of continued elaboration of concepts through the four years of the program. Each year of the program covers fewer topics than traditional programs, but covers them in greater depth. Reviewers found a strength of the program to be the opportunity provided to students to build understanding of mathematics as they work on fairly unstructured problems.

Assessment is an integral part of IMP. Students are provided diverse ways to demonstrate understanding, including homework, portfolios, presentations, reinforcement and extension problems, unit and semester exams, and discussion. A major challenge in the implementation of IMP is using alternative assessment instruments by teachers unfamiliar with these approaches.

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## Professional Development Resources and Program Costs

IMP provides support for teachers and districts through teacher workshops, a published curriculum that includes detailed and comprehensive teacher guides for each unit of instruction, and an extensive network of regional centers to assist districts in the implementation of the program. The "strategies" book (*Introduction and Implementation Strategies for the Interactive Mathematics Program*) provides extensive guidance for teacher-leaders and administrators on issues such as providing support for teachers and reaching out to parents, as well as information on implementation costs. The national IMImplementation Center provides overall guidance to schools and districts adopting the programs, including an outreach coordinator with a toll-free number, an electronic network to facilitate communication, an annual meeting for directors of IMP Regional Centers, and an annual meeting of teacher-leaders from around the country. Evaluation Updates are also available which summarize ongoing program evaluation efforts.

For each grade level, student textbooks cost \$37.95 per student. A teacher's guide to each of the 5 thematic units per year costs \$18.95 each (for a total of \$94.75 per course). Calculator guides for each of the four years cost \$12.95 each, and the teaching handbook for all four years costs \$9.95 each. However, all the teacher materials mentioned are available free with every set of 25 student books ordered. Graphing calculators and some manipulatives are also required.

Professional development is available through the regional centers. Contact the national implementation center for a listing of the centers, what professional development services they offer, and the cost of these services.

Where sufficient training and support are provided for teachers as they begin to teach this program, reviewers expect successful implementation. All units have extensive background and reference information to help teachers plan for and make instructional decisions. The most noted demand on the time and resources for a school will be the extensive training and preparation time suggested. Support is available through 11 regional centers offering flexible inservice workshops that can be arranged during the summer with follow-ups throughout the school year. IMP suggests that a school might reduce the load of teachers new to IMP to allow time for preparation and implementation of the new program. Reviewers commented that those schools willing and able to make a large investment in supporting the professional development of their teachers will be rewarded with high-achieving students able to think and reason mathematically.

IMP has been used in at least 24 states and 350 schools in diverse settings, including urban inner cities, rural farm areas, and suburban communities.

### *Educational Significance*

IMP's learning goals and subject matter are consistent with the NCTM standards. The program is designed to improve learning for a wide spectrum of students by offering extension activities for accelerated learners and reinforcement activities for students needing additional practice with the material.

## PROGRAM EFFECTIVENESS AND SUCCESS

The *Interactive Mathematics Program* has been designated an **Exemplary** mathematics program. IMP's research design included a five-year independent evaluation carried out by the Wisconsin Center for Education Research at the University of Wisconsin. A range of evaluation studies includes assessment of achievement in a variety of mathematical content areas, retention studies, the impact on continued study of mathematics, differential effects by ethnicity and gender, and changes in teaching practices.

In 1995, a matched-pair analysis of student scores on the mathematics component of the PSAT or SAT was done at three high schools from different parts of the country. In two of the schools, IMP students did better, to a statistically significant degree, than their peers. At the third school, IMP students scored higher, but the results were not statistically significant. Since then, at least 15 other schools have conducted their own SAT studies. In five of these schools, IMP students scored significantly higher than their traditionally taught peers and at the other ten schools there was no significant difference between the two groups.

A matched comparison study was done in 1996 to evaluate learning in the areas of statistics and problem solving. IMP students did significantly better than students in traditional programs both on the statistics items from the Second International Mathematics Study and on two problem-solving items developed by the state of Wisconsin. This study was replicated in two high schools in different parts of the country with matched samples from IMP and traditional classes in Spring, 1997, with similar results.

In 1997, a matched sample study compared the performance on the Stanford Achievement Test (SAT-9) of IMP and traditionally taught students at two high schools in Philadelphia. Results showed that the IMP students' scores were higher on 15 out

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of 20 reported SAT-9 multiple choice categories. However, only two of the 20 sub-scores were statistically significant, both in favor of the IMP students. These were in the areas of functions and probability.

In a transcript analysis study of more than 1,000 students enrolled at three California high schools during the years 1989–1993, students in IMP tended to take more mathematics classes and more semesters of college-qualifying mathematics than students enrolled in the Algebra-Geometry-Algebra II sequence. These differences were statistically significant for both female and male students and for African-American and White students (although students of all ethnic groups showed increases). The studies were conducted at a time when only the first three years of the IMP program were available to the schools participating in the study.

Changes in attitudes about mathematics and its application to daily life were documented in a separate 1993 study involving 450 students from the same three California high schools. In comparison to their peers in traditional programs, IMP students were more confident about their abilities in mathematics, more likely to see mathematics as meeting the needs of society rather than being a set of arbitrary rules, placed higher value on communication in mathematics learning, and were more likely to see mathematics applications in everyday life. All of these comparisons were statistically significant.

A study of about 240 teachers with IMP teaching experience was conducted in 1997. The teachers reported major changes in their practice in teaching mathematics, including their expectations about the depth of mathematics that students can learn, perceptions of their own and students' roles in the classroom, assessment of students' mathematical knowledge, the types of materials used in the classroom, the variety of mathematics that can be covered during the school year, the nature of preparation for class, and the use of cooperative learning strategies.

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## YEAR 1

### **Algebra**

Variables and Algebraic Expressions

Functions: Symbolic, Graphical, Situational, and  
Numerical

Signed Numbers

Linear Equations

### **Geometry**

Angles and Their Measurement

Angles of Polygons

Similarity and Congruence

### **Trigonometry**

Right-Triangle Trigonometric Functions

### **Probability and Statistics**

Theoretical and Experimental Probabilities

Expected Value

Normal Distributions and Normal Curve

Mean and Standard Deviation

### **Logic**

Conjectures and Counterexamples

Sound Logical Arguments

## YEAR 2

### **Algebra**

Equivalent Expressions

Distributive Property

Linear Equations and Inequalities in One and  
Two Variables

Dependent, Inconsistent, and Independent Pairs  
of Linear Equations

Linear Programming Problems with Two Variables

Laws of Exponents

Scientific Notation

### **Logic**

Indirect Proof and Proof by Contradiction

If, If Then Statements

### **Geometry**

Areas of Polygons

Surface Area and Volume for Three-Dimensional  
Solids

Pythagorean Theorem

Relationship Between the Areas and Volumes  
of Similar Figures

### **Trigonometry**

Right-Triangle Trigonometry

### **Probability and Statistics**

Statistical Experiments

Hypotheses Testing

Chi-Square Statistic

Statistical Significance

Conditional Probability

# INTERACTIVE MATHEMATICS PROGRAM: Table of Contents



# INTERACTIVE MATHEMATICS PROGRAM: Table of Contents

## YEAR 3

### **Algebra**

Quadratic Equations  
Completing the Square  
Exponential and Logarithmic Functions  
Elimination Method for Solving Systems of Linear Equations  
Matrix Algebra  
Linear Programming in More Than Two Variables

### **Analytic and Coordinate Geometry**

Slope and Rate of Change  
Distance and Midpoint Formula  
Distance from a Point to a Line  
Equations of Planes

### **Precalculus**

Inverse Functions  
Derivative

### **Geometry**

Area and Circumference of a Circle  
Equation of a Circle  
Perpendicular Bisectors and Angle Bisectors  
Intersections of Lines and Planes

### **Trigonometry**

Right-Triangle Trigonometry

### **Probability and Statistics**

Combinations and Permutations  
Pascal's Triangle  
Binomial Distribution

### **Logic**

If and Only If Statements  
Converse of a Statement

## YEAR 4

### **Algebra**

Quadratic Formula and Its Proof  
Physical Laws of Falling Bodies in Terms of Quadratic Functions

### **Analytic and Coordinate Geometry**

Polar Coordinates  
Matrix Representations of Geometric Transformations  
Analytic Expression for Projection onto a Plane from a Point Perspective  
Representing a Line in 3-Space Algebraically

### **Precalculus**

The Effect of Changing Parameters on Functions  
Asymptotes of Rational Functions  
Algebra of Functions, Including Composition and Inverse Functions  
Least-Squares Approximation and Curve-Fitting

### **Trigonometry**

Circular Functions  
Radian Measure  
Trigonometric Identities

### **Probability and Statistics**

Binomial Distribution  
Central Limit Theorem  
Confidence Interval, Confidence Level  
Margin of Error

### **Programming**

Program Creation and Interpretation  
Animation on a Graphing Calculator



*Everyday*

PROMISING PROGRAM

# MATHEMATICS

# EVERYDAY MATHEMATICS

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

*Everyday Mathematics* is a comprehensive and balanced mathematics program for Grades K–6. This curriculum teaches basic computational and arithmetic skills, as well as a broad range of mathematical concepts including data and probability; geometry and spatial sense; measures and measurement; numeration and order; operations; patterns, functions, and sequences; and reference frames. Algebra and uses of variables begin in the third grade.

Mathematics is also integrated with other content areas and with real-life situations. This enriched curriculum includes such features as problem solving about everyday situations; linking past experiences to new concepts; sharing ideas through discussion; developing concept readiness through hands-on activities and explorations; cooperative learning through partner and small-group activities; and enhancing home-school partnerships.

## PROGRAM QUALITY

The program's goals, based on the NCTM standards, were found by reviewers to be explicit, comprehensive, and challenging. The goals, connected both to other areas of mathematics and to other disciplines, are clearly outlined at the beginning of each lesson. In addition, students are presented with clear goals and expectations in the student journal. Consistently building on previous understandings, the program emphasizes depth of understanding through the use of varied, meaningful, and real-life activities. The program's content promotes student communication, both oral and written, and is consistently aligned with its learning goals.

The program challenges students to think about how they can apply concepts they have learned to everyday situations, and it emphasizes understanding and application. Students are asked to synthesize their learning and develop their own strategies for solving problems. The instructional design

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## Professional Development Resources and Program Costs

For each grade level, the program includes a Teacher Resource Package and student materials. The Teacher Resource Package includes: *Teacher's Manual and Lesson Guide*, *Teacher's Resource Book of Blackline Masters*, *Teacher's Reference Manual* (which contains comprehensive background information about mathematical content, teaching strategies, and classroom management), *Assessment Guide*, *Scope and Sequence Chart*, a guide to Creating Home and School Partnerships and student materials (consumable journals and activity books for Grades 1–3, consumable journals and non-consumable reference materials for Grades 4–6). Other materials for the program include manipulative kits, a staff development kit, and a technology guide.

Staff Development Videos can be obtained for Grades 1–5. These videos provide an in-depth look at the *Everyday Mathematics* curriculum, showing a model lesson as well as a look at the content covered in each unit. A Grades K–6 overview

Video for Teachers and a Parent Overview Video are also available. Each video costs \$50. An *Everyday Teaching Staff Development Kit* provides seven mathematics content workshops designed to be conducted by in-school curriculum specialists, principals and classroom teachers.

Users' Conferences are offered every summer at various locations. In addition, pre-service and in-service training can be provided at the district level. In-service training is facilitated by experienced *Everyday Mathematics* teachers and administrators.

Teacher Resource Packages range from \$120 for Kindergarten to \$170 for Grade 6. Student Materials range from \$14.95 per student to \$25 per student for Grades 1–5. Classroom manipulative kits for each grade level are available and range from \$245 (Kindergarten) to \$430 (Grades 1–2).

links past experiences with exploration of new concepts. Varied approaches to instruction address different learning styles as well as abilities.

The assessment system is varied and comprehensive. The routine assessments are designed to track student progress and direct individual student learning. There are numerous options in addition to the daily routines in the teacher manuals and student materials. For example, there are unit assessments, mid-year and end-of-the-year assessments, journal writing, projects and explorations, and written and oral assessments. Because of the diversity of activities offered, students can demonstrate depth, flexibility and application of learning. The assessment handbooks provide rubrics and guidance enabling teachers to adopt assessments to suit their environment.

## PROGRAM EFFECTIVENESS AND SUCCESS

*Everyday Mathematics* has been designated a **Promising** mathematics program. Positive program effects on students' mathematics achievement are documented in the "Success Stories," which are snapshots of evidence rather than comprehensive studies. However, the data were compiled from eight diverse communities in Illinois, Michigan, Pennsylvania, Texas, and Washington and the "*Everyday Mathematics* Student Achievement Studies."

In 1995 and 1996 in the Kalamazoo, Michigan, Public School district, positive results were seen for students, including an increase from 41% in 1994 to 54% in 1996 in the percentage of fourth graders who passed the Michigan statewide achievement tests (MEAP). Additionally, the number of schools that scored satisfactory and above on the mathematics assessment increased from 2 of 18 in 1994 to 8 of 18 in 1996. Significance tests were not conducted. In 1995, the highest increases in percentages of students scoring satisfactory and above occurred at three elementary schools with the highest number of low-income students (80%+ eligible for free or reduced-price lunch). From 1991 to 1996, third grade scores on the

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## Usefulness to Others

*Everyday Mathematics* is a flexible curriculum that provides training material to allow any school to adopt this program. It is currently used by over one million students nationwide in schools and districts of various sizes and diverse locations.

## Educational Significance

Each level of the program is a year-long, comprehensive curriculum that correlates highly with NCTM standards. There is an in-depth study of each strand of mathematics presented in the program. Students are expected to understand concepts as well as develop knowledge.

The program allows for flexibility, both in the overall implementation of the program and in strategies for assessing students. The materials recognize and address all learning styles, use real-world contexts, and encourage students and teachers to value and enjoy mathematics. As a result, the program provides opportunities for children of all ability levels to succeed at learning.

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Illinois Goal Assessment Program (IGAP) at a Chicago school increased from 180 to 319 (scale 0 to 500), well above the city's average score of 221 and the state's average score of 286, which was statistically significant. A study conducted in one Washington state school in 1993–1994 using the Test of Problem Solving demonstrated statistically significant gains in mean performance by *Everyday Mathematics* students.

In addition, independent researchers at Northwestern University are carrying out a major longitudinal study. The preliminary results from these studies document significant positive program effects on student achievement. An early report from this study (Student Achievement Studies) shows that *Everyday Mathematics* first graders scored 9% to 21% higher than traditionally taught U.S. students and were comparable to an international student sample on computation, word problems, multidigit equations, estimations, and mental computation. The most recent report from this longitudinal study, in regard to fifth grade student achievement, indicates that *Everyday Mathematics* students scored above the U.S. comparison students, to a statistically significant degree, on 76% of the questions and below them on none. Questions covered the topics of mental computation, division situations, number sense, algebra knowledge, and geometry and measurement knowledge. Citations from teachers in the "Success Stories" and examples of student work demonstrated students' use of problem solving and reasoning and provided insight into what the students were expected to do.

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# EVERYDAY MATHEMATICS: Table of Contents

## **Kindergarten**

Exploring Data and Chance  
Geometry and Spatial Sense  
Measures and Measurement  
Numeration and Order  
Operations  
Patterns, Function, and Sequences  
Reference Frames

## **Grade 1**

Establishing Routines  
Everyday Uses of Numbers  
Visual Patterns and Number Patterns  
Thermometer, Linear Measures, and Clocks  
Relations, Number Stories, and  
Mental Arithmetic  
Introduction to Fact Power  
Geometry and Attributes  
Mental Arithmetic, Money, and Fractions  
Numeration and Place Value  
End-of-Year Reviews and Assessments

## **Grade 2**

Routines and Assessments  
Addition and Subtraction Facts  
Place Value, Money, and Time  
Mental Arithmetic: Addition and Subtraction  
3-D and 2-D Shapes  
Review and Extension of Whole-Number  
Operations  
Patterns and Rules  
Fractions  
Measurement  
Decimals and Place Value  
Whole-Number Operations Revisited  
Year-End Reviews and Extensions

## **Grade 3**

Routines, Reviews, and Assessments  
Adding and Subtracting Whole Numbers  
Linear Measures  
Multiplication and Division  
Place Value in Whole Numbers and Decimals  
Geometry  
Multiplication and Division  
Fractions  
Multiplication and Division  
Measurement  
Probability; End-of-Year Review

## **Grade 4**

Naming & Constructing Geometric Figures  
Using Numbers & Organizing Data  
Multiplication & Division; Number Sentences  
and Algebra  
Decimals & Their Uses  
Map Reference Frames; Measures of Angles  
Fractions & Their Uses; Chance & Probability  
Big Numbers, Estimation, & Computation  
Reflections & Symmetry  
Percents  
Mammals: An Investigation  
Perimeter & Area  
Rates  
3-Dimensional Shapes & Volume

## **Grade 5**

Number Theory  
Estimation & Calculation  
Geometry Explorations & the American Tour  
Fractions, Decimals, & Percents  
Collecting & Working with Data  
Division, Ratios, & Exponential Notation  
Coordinates, Areas, & Circles  
Algebra Concepts & Skills  
Fractions & Ratios  
Volume  
Fractions, Multiplication, Percents, & Rates

## **Grade 6**

Collection, Display, & Interpretation of Data  
Rational Number Uses & Operations  
Scientific Notation & the Solar System  
Variables, Formulas, & Graphs  
Geometry: Congruence, Constructions, &  
Parallel Lines  
Rates & Ratios  
Probability & Discrete Mathematics  
Number Systems & Algebra Concepts  
More about Variables, Formulas, & Graphs  
Geometry Topics

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PROMISING PROGRAM

# MATHLAND

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# MATHLAND

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

*MathLand* is a comprehensive mathematics program for Grades K–6 focused on students' use of problem-solving approaches to investigate and understand mathematical content.

The program is based on a 36-week school year and is divided into 10 units at each grade level, with the exception of kindergarten, which has 8 units. *MathLand* presents students with coherent, connected, and accessible mathematical experiences. The foremost learning goal for *MathLand* is that students become mathematically proficient and confident problem-solvers. Each unit of the program includes investigations, problems, and exercises that are interrelated. The materials emphasize development of skills such as problem solving, communication, reasoning, mathematical connections, estimation, measurement, statistics, probability, fractions and decimals. Progressing through the years, students develop both basic skills and advanced conceptual understanding and problem-solving skills through the use of *MathLand* materials.

## PROGRAM QUALITY

*MathLand* is based on the NCTM standards. The program's goals are challenging, clear, and appropriate for the intended student population. The program was designed to involve students in hands-on investigations to foster problem solving, mathematical communication, connections between disciplines and the real world, and reasoning skills. Concepts are studied in depth, allowing for skill development through exploration and activities, discussion, collaborative work, the use of hands-on materials, technology, graphical and pictorial displays, dramatizations, and oral and written presentations.

The instructional design provides student involvement through open-ended activities for which the teacher becomes the coach, prompting student discussions. *MathLand* uses students' everyday life as the context for activities, beginning with students' everyday language and connecting it to mathematical language and symbols.

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## Professional Development Resources and Program Costs

The teacher's *Guidebook* provides instructions for the units and investigations, and the *Assessment Guide* outlines a philosophy of assessment and assessment options, as well as support activities (contained in *Daily Tune-Ups*) that describe skills development, reinforcement, and review activities for engaging the whole class. *Skill Power*, a reproducible or consumable book for students, offers daily computation and problem-solving practice. The teacher's *Resource Manager* contains reproducibles and supportive material to help teachers understand unit goals and concepts, the use of assessment in the classroom, and implementation adaptations designed for second-language learners, diverse learning styles, and multi-age classrooms. Materials to share with parents are also included.

When a school or district adopts *MathLand*, a professional development program will be designed to meet the needs of the school or district. Staff development is provided by classroom teachers who have in-depth experience with *MathLand*.

*MathLand* student materials include *Skill Power*, which is purchased for every student for \$6.95 per year. *ArithmeTwists* can be purchased for each student as an enhancement for \$5.95 per book. Teachers' print materials and classroom manipulative kits are purchased only once, during the initial year of implementation. These cost approximately \$1,250 per class for teacher's guides and consumable, non-consumable, and overhead manipulatives.

The multifaceted approach to assessment in the program allows the teacher to make informed judgment about student learning. Observation, portfolios, pre- and post-tests, daily check-ups, and journal writing give a clear picture of student progress. Scoring rubrics are provided based on the curriculum to help determine the degree of student understanding and enable the teacher to give feedback to students and parents in a meaningful way.

## PROGRAM EFFECTIVENESS AND SUCCESS

*MathLand* has been designated a **Promising** mathematics program. Overall, the combined results from several studies revealed positive trends in student achievement in a variety of educational settings with multiple student populations.

Student performance data collected in the 1996–1997 school year from 31,824 students in the San Francisco Unified School District using the California Test of Basic Skills demonstrated that *MathLand* students scored higher on math, by a statistically significant degree (more than one year's growth), than the norm group. Also, data collected from third and fourth grade students in six Los Angeles Unified School District schools using a traditional standardized test demonstrated that *MathLand* students scored higher to a statistically significant degree (in excess of one year's growth) on mathematics application than the norm group. *MathLand* students achieved one year's growth in computation.

Consistently positive results were also found in a 1996–1997 study using the Terra Nova Multiple Assessment. Overall, 4% more *MathLand* students achieved mastery than the national norm group on the problem-solving and reasoning objectives. In addition, overall, 2% more *MathLand* students demonstrated mastery than the national norm group on the objectives of number and number relations; 7% more *MathLand* students demonstrated mastery on computation and numerical estimation; and 5% more *MathLand* students demonstrated mastery on geometry and spatial sense. An equal percentage of *MathLand* students and the national norm group

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## Usefulness to Others

Published in 1994, *MathLand* has been used since 1995 in districts with highly diverse student populations, including San Francisco, Los Angeles, Boston, and in Department of Defense Dependents' Schools, Title I schools, and multilingual/bilingual classrooms.

The *Guidebook* provides clear, detailed instructions for all units and investigations, as well as encouraging notes of advice drawn from research and classroom experiences. While an easy-to-follow scope and sequence chart is included along with the materials needed for each lesson, reviewers noted that teachers may need additional planning time, at least initially.

## Educational Significance

Using a child-centered approach, *MathLand* is designed to improve learning for a wide spectrum of students. Reviewers noted that the subject matter is appropriate for students at each grade level, and the activities appear sufficiently varied to hold student interest. The program's strength lies in its coverage of mathematical content and its attention to conceptual understanding, communication, reasoning, and problem solving.

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mastered the objectives of data analysis, statistics, probability, and communication. No significance tests were conducted.

Eight districts participating in *MathLand*'s Assessment Consortium joined together to conduct a survey in 1996–1997 to study student achievement during the first year of *MathLand* implementation. Existing data were collected from these eight districts at Grades 2–6, representing almost 30,000 students, using district-administered standardized tests. The data represented a variety of traditional standardized tests. External analysis of the Normal Curve Equivalent (NCE) scores showed more than one year's growth, a statistically significant difference, in mathematical application for *MathLand* students in three of the eight districts and one year's growth in the other five districts.

### **Kindergarten**

All About Me: Data Analysis  
 Red, Red, Red: Patterns  
 Beginning Numbers: Beginning Numbers  
 Making Groups: Logical Thinking  
 Let's Count: Numbers to 10  
 Shapes: Geometry  
 Long and Short: Measurement  
 Numbers Big and Small: Number Concepts

### **Grade 1**

All About Us: Data Analysis  
 Snap, Clap: Patterns  
 How Many?: Numbers to 20  
 Collections and Questions: Logical Thinking  
 Number Combinations: Number Relations  
 Making Comparisons: Measurement  
 Equations: Addition/Subtraction  
 Tens and Extras: Tens and Extras  
 Seeing Shapes: Geometry  
 Take a Chance: Probability

### **Grade 2**

All About Us: Data Analysis  
 Equations: Addition/Subtraction  
 Making Predictions: Patterns and Functions  
 Strategies: Strategies  
 Collections and Questions: Logical Thinking  
 Making Groups: Number Relations  
 How Long? How Heavy?: Measurement  
 Expanding Number Relations: Expanding Numbers  
 Shaping Questions: Geometry and Visual Thinking  
 Anything's Possible: Probability and Statistics

### **Grade 3**

All About Us: Data Analysis  
 Skip-Counting Patterns: Patterns  
 Strategies: Strategies  
 Equal Groups: Multiplication  
 Creative Feature: Logical Thinking  
 Numbers Beyond and Between: Number Relations  
 Approximations: Measurement  
 Getting Squared Away: Geometry  
 Animals, Animals: Applications  
 A Fair Shake: Probability

### **Grade 4**

All About Us: Data Analysis  
 Looking For Rules: Patterns  
 Strategies: Strategies  
 That's Unique: Logical Thinking  
 Numbers Between Numbers: Fractions  
 Spaces: Measurement  
 Representations: Multiplication/Division  
 Shaping Relationships: Geometry  
 Dinosaur Days: Applications  
 'Round They Go: Probability

### **Grade 5**

All About Us: Data Analysis  
 Using Predictions: Patterns and Functions  
 Strategies: Number Relations  
 Organizations: Logic and Algebra  
 Millions: Number Relations  
 Inside/Outside: Measurement  
 Equivalence Thinking: Fractions  
 The Triangle Angle: Geometry and Visual Thinking  
 Systems Thinking: Applications  
 Confidence: Probability

### **Grade 6**

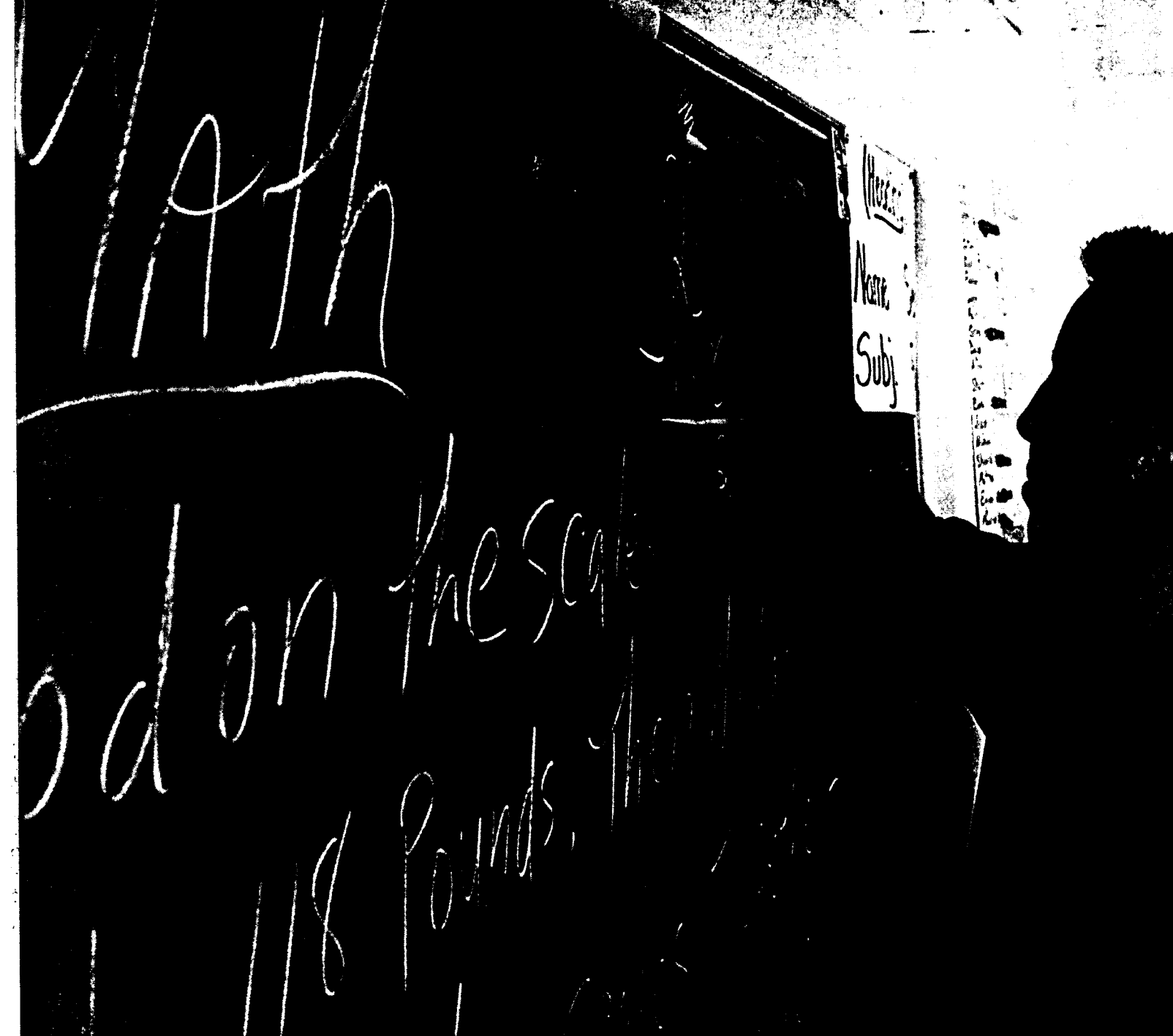
The Average Student: Data Analysis  
 Algebra Thinking: Algebraic Thinking  
 Strategies: Strategies  
 Numbers Between Numbers: Fractions/Decimals  
 Rules and Relationships: Primes/Percents/Integers  
 Sizes of Shapes: Measurement  
 Doubling Discoveries: Patterns  
 Round and About: Geometry  
 City Changes: Applications  
 Equal Chances: Probability

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*Middle-school*

PROMISING PROGRAM

**MATHEMATICS**

*through Applications Project (MMAP)*

# MIDDLE-SCHOOL MATHEMATICS THROUGH APPLICATIONS PROJECT (MMAP)

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

*Middle-school Mathematics through Applications Project*, MMAP, is a middle school mathematics program for students in Grades 6–8. It is a series of technology-integrated, project-based units that fit together into a comprehensive middle school mathematics curriculum that emphasizes proportional reasoning and algebra/functions. It also covers statistics, probability, measurement, and geometry. MMAP materials engage students in using technology to create and analyze solutions to real-world problems. The materials are designed so that students will increase their conceptual understanding of mathematics, and will develop competence with standard symbolic notations for mathematical concepts and an ability to organize their mathematical work in a way that adequately communicates their ideas.

The MMAP curriculum comprises *Application Units*, *Extensions*, and *Investigations*. The *Application Units*, accompanied by software, immerse students in an extended role-play in which mathematical concepts and skills are integrated and developed along the way. The design context provides reasons and resources for developing the mathematics. *Extensions* build directly on the mathematical insights and skills students develop within the *Application Units*. *Investigations* introduce other mathematical concepts within creative contexts or from a pure mathematics point of view. Mathematical concepts, common notations, and vocabulary are emphasized, along with mathematical practices such as making conjectures, devising counter-examples, and writing proofs. All of the materials address the skills of problem solving, communication and reasoning.

Strengths of MMAP include integrated use of technology in the core curriculum, adaptability to local and state curriculum expectations, an emphasis on real-world contexts and work-place simulations, and the ability to meet students at their skill level and move them forward. Each of the four interactive software components provides mathematical resources and data for analysis for the group-based projects.

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## Professional Development Resources and Program Costs

Teachers are required to attend initial and annual training at a fee of \$150 per teacher per year. Ongoing training is also provided through the use of videotapes and interactive materials. Technical support is available on-line and through a Customer Service Hot Line.

MMAP will be published by Voyager Expanded Learning and will be available in September 1999. Pricing for MMAP is based on an annual licensing fee starting at \$2,580 per class of up to 30 students. The licensing fee covers the teacher's Curriculum Guide, computer software, and essential consumable and non-consumable materials that provide five hours of instruction per week.

Students will use computers, calculators, and other math tools. Software will be available in both Macintosh and Windows versions. The units can be completed with a single computer in the classroom and/or with access to a computer lab. Costs for computers need to be factored into the total cost of implementing the program. Scientific calculators are recommended.

## PROGRAM QUALITY

Reviewers found that MMAP's goals are aligned with NCTM standards. The goals emphasize depth of understanding, reasoning, communication, problem solving, and connections through the application of mathematics skills, knowledge and understandings. The units, enhanced by the use of computers, are centered around proportional reasoning and algebraic thinking, but patterns, functions, algebraic concepts, computation and estimation are all included as important areas. Students progress from sixth through eighth grade along a number of mathematical dimensions. Supplemental mathematics activities are provided to help students master necessary skills. Each of the units involves problem exploration using a range of graphical, numerical, and algebraic representations.

The instructional design of the program is creative, flexible, appealing, and motivating both to students and teachers. The selection of problems and the inclusion of extension and investigation modules are helpful, and are appropriate and challenging mathematics for middle school students. Scenarios in which students learn in context provide purpose for learning rigorous mathematics and are one of the program's strengths.

The application unit provides the motivation to get students engaged in the learning process. Confronted with a problem, students work in small groups to create and analyze designs with the aid of computer software, make conjectures, and present solutions. Students write about what they learn, reflect on their activities, assess each other and model situations. Reviewers noted that the activities provide multiple avenues for connections within mathematics itself and with other subject areas (for example writing, geography, and science), and therefore attend to a broad range of student interests.

Reviewers found MMAP's approach to assessment to be one of the most appealing features of the program. Flexibility in the on-going assessment process is encouraged, allowing teachers to gear their assessments to meet the individual needs of their students. The assessment process stresses the importance of a variety of means of communicating student understanding, and a variety of options exist to

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## FOR FURTHER INFORMATION CONTACT:

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## Usefulness to Others

MMAP offers a comprehensive middle school curriculum that can be successfully transferred to other settings. It is currently in use in racially, economically, and linguistically diverse communities in California, Alaska, New York, Florida, Michigan, Oregon and Washington. It also serves students with learning disabilities.

This program will be most successfully implemented in schools where students have constant access to a set of mathematical tools, including calculators and measuring devices, and where each MMAP class has access (in the classroom or computer lab) to at least 4 to 6 computers for 60% of the *Application Unit*. MMAP is currently available only for Macintosh computers, but Windows versions will be available in the fall of 1999. Spreadsheet and word processing programs are recommended. Access to the Internet is suggested for some units. Some units will have a "low tech" path that will use calculators.

Workshops on using computers/software are recommended for teachers. Training on managing groups in the classroom and on the assessment methods recommended by the program are also encouraged. A Web site has been established for teachers.

## Educational Significance

The MMAP program is well designed and addresses several current needs in mathematics, including connecting mathematics to the real world and useful assessment strategies. The modular structure allows the instructor to address his/her students' individual needs. Real world applications, interdisciplinary investigations, and the use of technology to enhance learning capture students' interest and attention. Students have opportunities to develop important mathematical and reasoning skills that will carry over into higher mathematical classes.

support this, such as journals, log books, presentations, writing, peer reviews, teacher-student conferences, group meetings, and others. Progress reports can be given almost daily, thus allowing the student immediate feedback.

The program's design allows teachers greater-than-average time to observe students in their learning. The "assessment tips" section provided in many activity sections also aids teachers. These tips include assessment goals and a sample Assessment Map with suggestions to help teachers make decisions about which activities may be useful.

## PROGRAM EFFECTIVENESS AND SUCCESS

*Middle-school Mathematics through Applications Project* (MMAP) has been designated a **Promising** mathematics program. There is preliminary evidence, often data collected from individual classroom-based evaluations or teacher reports, that indicates MMAP has positive effects on student performance and attitudes in some areas of mathematics study, such as problem solving.

One study conducted in a San Francisco classroom reported that MMAP students made gains in mathematics achievement, especially for students who scored in the 70<sup>th</sup>–80<sup>th</sup> percentile range the previous year. In addition, data from a 1994 study, obtained from 24 teachers and 42 students who responded to written questionnaires and several teachers who participated in individual interviews, demonstrated positive effects on MMAP students' skills in both the cognitive and affective domains.

Preliminary results reported in 1998 of a continuing comprehensive study utilizing teacher interviews and assessments, classroom observations, and teacher and student interviews confirmed the emerging pattern of positive outcomes. Data indicated that MMAP students see math as a tool, rather than a subject; special education students can participate in MMAP and accomplish tasks with equal status as regular and advanced math students; and students of diverse economic and racial backgrounds have equal access to the contextual knowledge necessary for completing MMAP units. A comprehensive study is underway.

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## TO ORDER CONTACT:

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1125 Longpoint Avenue  
Dallas, TX 75247

Telephone: (888) 399-1995

Fax: (214) 631-0176

E-mail: [shansen@iamvoyager.com](mailto:shansen@iamvoyager.com)

WWW Home Page: <http://www.iamvoyager.com/>



## 6th Grade:

### Plan an Event

computation and estimation

### Displaying Data

explore problems using different graphical models;  
interpret in original problem situation

### Geometry in Quilting

geometric shapes; properties and relationships

### Dream Home

scale as a proportion; 2-D and 3-D measurement

### Measurement Systems

understand the structure of systems of measurement

### Problems with Proportions

proportional reasoning; equivalent fractions and cross-multiplying

### Codes Inc, Compression

percent growth or shrinkage; calculating and understanding percents

### Investigating Percents

definition and application of percents  
multiple representations ñ fractions, decimals

### A Rational Expedition

operations of and representations of rational numbers

### Fractions 101

develop, analyze, and explain procedures for computation with fractions

### Guppies

using combinations of rates; linear and exponential functions; percents and decimals

### From Patterns to Functions

describe and represent patterns with tables, graphs, and rules

### Algebra Vocabulary

understand algebra vocabulary such as variable, coefficient, exponent, term, expression, constant, binomial, and trinomial

## 7th Grade

### The Antarctica Project

scale as proportion; linear function; direct and inverse variation; independent and dependent variables; area and perimeter relationships

### Connecting to Algebra

generalize patterns; variable, constant, and equation work

### Direct & Inverse Variation

understand direct and inverse variations as tables and graphs

### Working with Positive and Negative Numbers

develop number sense for positive and negative numbers and operations on them

### Functioning in the Real World

multiple representations of functions; tables, charts, graphs, description in words; rate, time, distance

### From Patterns to Functions

describe and represent patterns with tables, graphs, and rules

### Probability

sample space; theoretical and experimental probability

### Mobile Math

construct, sort and classify 2-D and 3-D shapes; identify their properties; geometry vocabulary

### Codes Inc., Privacy

algebraic expression of linear and non-linear functions; finding algebraic patterns from number patterns; number theory: factors and multiples

### Coding with Number Theory

number theory; modular arithmetic; prime numbers

### Excursions

scale and slope; variables; discrete math; geometry

### Pythagorean Theorem

explore applications of the Pythagorean Theorem in a variety of situations



MIDDLE-SCHOOL MATHEMATICS THROUGH APPLICATIONS PROJECT:  
*Table of Contents*

**8th Grade**

**Environmental Justice**

scale as unitless ratio; algebraic inequalities; logic and sets

**Lottery Likelihoods**

fairness; determine probabilities; experimental vs. theoretical probability

**Equivalent Expressions**

understand distributive and cumulative properties; use algebraic notation to describe patterns and relationships

**Binomials and Trinomials**

use and develop methods for multiplying binomials; gain more understanding for algebraic notation

**Traffic Light Math**

logic

**Counting in the Wild: Sampling**

estimation; using formulas

**Wolves and Caribou**

using combinations of rates to get a new rate; linear and exponential functions; recurrence functions; multi-function systems; modeling; combinatorics; calculating and interpreting percents and decimals

**Functions From Various Views**

multiple representations; understand functions as tables, graphs, and rules

**Review**

**Prove It**

**Codes on the Internet**

properties of functions; combinatorics





# *Number* POWER

PROMISING PROGRAM

# NUMBER POWER

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

*Number Power* is a supplemental mathematics program for Grades K–6 that focuses on developing number sense and social interaction skills in cooperative settings. It is designed to work as a support or replacement for number concepts and operations sections of whichever mathematics series is used in the classroom.

In Grades K, 1, 4, 5, and 6, the curriculum consists of approximately 30 multi-day lessons organized into three units. In Grades 2 and 3, the curriculum consists of approximately 60 multi-day lessons organized into seven units. These units engage students in cooperative group work as they learn to make sense of numbers and their relationships, explore numbers as quantities, make judgments about the reasonableness of numbers and solutions, and build understanding about the effects of operations on numbers.

The complexity of the number sense concepts varies from “describe the relative magnitude of numbers by comparing the sizes of different groups of objects” in Kindergarten, to “describe the relative magnitude of fractions, percents, decimals, and ratios” in 6th grade. Students are first introduced to a question or concept, and then they explore the concept in different situations over a period of time. Throughout, students learn to estimate, analyze data, use mental computation, and devise their own computation and problem-solving strategies as well as gain proficiency in accurate and efficient computation. They are also encouraged to make connections to their prior learning.

## PROGRAM QUALITY

The goals of the *Number Power* program are consistent with NCTM standards, emphasizing development of number sense in students, in Grades K–6, in cooperative settings. The goals are challenging, clear, and appropriate for the intended student populations. Through the use of open-ended mathematical problems as well as real-life experiences, students actively engage in activities (such as collecting, organizing, and analyzing data) that develop skills in critical thinking, reasoning about numbers, reflection, and problem solving. The lessons are of appropriate length to provide time for reflection and understanding. Additionally, the content of the program makes connections within

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## Professional Development Resources and Program Costs

Professional development is available at \$750 per day plus materials and travel expenses. The length of staff development varies, and it is customized to meet the client's needs, ranging from a one-day overview workshop to in-depth institutes and on-site coaching and support over a period of months or years.

Teacher materials consist of one *Teacher Resource Book* for each of Grades K, 1, 4, 5, and 6, and two *Teacher Resource Books* for Grades 2 and 3. The instruction pages of each book describe the design of the lesson and the materials required and include all necessary blackline masters. These

books also provide information on the program's pedagogy and management and address how to assess students' mathematical thinking and how to plan and modify instruction to optimize student learning. Additional planning time may be needed in the first year to create and organize the materials.

The *Teacher's Resource Book* costs \$19.95 each. Some common objects, such as beans or magazine pages, may have to be collected, and some materials, such as handouts, spinners or money, may have to be created or reproduced. Most of these items could be reused after the first year.

mathematics and between mathematics and other content areas. The open-ended nature of the activities offers access to problems and learning opportunities for all students.

Reviewers found the program's instructional design appropriate, engaging, and potentially motivating for students. References to prior knowledge and commonly held conceptions were frequently made, and students were able to collaborate in groups around problems or situations relating to their interests. *Number Power* emphasizes depth of understanding and is designed so that students build upon what they know, applying new concepts to new situations. The lessons support student thinking and evoke multiple ways of solving problems. The design reflects current research on how children learn numbers, learning styles, and cooperative grouping.

The ongoing, informal approach to assessment is designed to guide student learning and inform teacher planning. There are multiple methods, opportunities, and suggestions for ongoing assessment throughout the program. Students are also involved in the assessment process through reflection and self-assessment. Assessment is open-ended and was found to be appropriate and adequate in informing student learning and guiding teachers' instructional decisions. Teacher questioning strategies and notes about teacher observations are written in detailed form in each lesson.

## PROGRAM EFFECTIVENESS AND SUCCESS

*Number Power* has been designated a **Promising** mathematics program. A 1993–1995 evaluation of the effects of the program on teachers and students in second through fifth grade classrooms was conducted by the developer of *Number Power*. Six elementary schools in the San Francisco Bay area were randomly assigned to each of three conditions: curriculum only, curriculum plus staff development, and comparison. Teachers at the curriculum only schools received the *Number Power* curriculum books and accompanying resource materials. Teachers at the staff development schools received six days of workshops and from 0 to 7

*Number Power* is designed to promote success of students from a broad range of cultural, racial, socioeconomic, and academic backgrounds. Concrete examples and opportunities to collaborate are particularly helpful for students who are learning English as a second language. The program is currently in use nationwide and in Canada and Australia. In the U.S. it has been utilized in urban and suburban areas and among diverse populations.

## Educational Significance

*Number Power* emphasizes the use of reasoning about numbers to solve mathematical problems. This includes selecting and using computation techniques as appropriate to specific problems and then determining when a result makes sense. As a supplement or a replacement to parts of a core math curriculum, it helps students develop flexibility with numbers, understand number meanings and multiple relationships among numbers, and recognize the magnitude of numbers. It also emphasizes learning as a cooperative process and provides students the opportunity to work with partners and in small groups and to reflect on and discuss problems involving how they worked together.

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days of individualized follow-up coaching over the two years of the evaluation in addition to the curriculum and resource materials. Teachers in the comparison condition continued to teach their "regular" mathematics curriculum during the evaluation.

A total of 51 teachers participated in the evaluation; 234 students were interviewed during the first evaluation year and 228 during the second evaluation year. The interviews were conducted using two semi-structured interviews developed for the evaluation, one for use with second and third grade students focusing on whole number operations, and one for fourth and fifth grade students focusing on rational number operations. Students were asked to perform mathematical operations and then questioned on their approach.

In contrast to the comparison group, both the curriculum only and the curriculum plus staff development groups had significantly greater growth over time (than the comparison groups) in terms of (1) students' depth of mathematical understanding, (2) students' number sense, and (3) flexibility of approach to problem solving. No statistical differences were found in students' clarity of communication or students' confidence in their mathematical abilities.

# NUMBER POWER: Table of Contents

## **Kindergarten**

Exploring Numbers  
Exploring Numbers and Operations  
Exploring Computation and Problem Solving

## **Grade 1**

Exploring Numbers and Operations  
Exploring Relative Magnitude  
Exploring Number Composition

## **Grade 2, Volume 1**

Grouping and Number Patterns  
Exploring Tens and Hundreds  
Sorting and Informal Computation

## **Grade 2, Volume 2**

Number Meaning and Relationships  
Mental and Informal Computation  
Informal Computation

## **Grade 3, Volume 1**

Exploring Tens and Hundreds  
Mental Computation  
Data Analysis and Number Sense

## **Grade 3, Volume 2**

Computing with Multiples of Ten  
Mental Computation and Number Relationships  
Informal Computation with Hundreds

## **Grade 4**

Fraction Explorations  
Fractions: Parts and Wholes  
Large Numbers

## **Grade 5**

Comparing and Finding Equivalents  
Connecting Fractions and Percents  
Fractions, Percents, and Decimals

## **Grade 6**

Music Statistics, Percents, and Ratios  
Exploring Ratio Relationships  
Using Ratios



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*The University* PROMISING PROGRAM  
*of Chicago School*  
**MATHEMATICS**  
*Project (UCSMP)*

# THE UNIVERSITY OF CHICAGO SCHOOL MATHEMATICS PROJECT (UCSMP)

## PROGRAM DESCRIPTION AND CURRICULUM MATERIALS

*The University of Chicago School Mathematics Project* (UCSMP) secondary level materials were first developed in the years 1983–1991 and then revised over the years 1993–1998. These materials constitute a six-course mathematics curriculum, each lasting a full-year, for students in Grades 7–12. The six courses offered are: *Transition Mathematics*; *Algebra*; *Geometry*; *Advanced Algebra*; *Functions, Statistics, and Trigonometry*; and *Precalculus and Discrete Mathematics*. Each course is intended to stand alone, but it is preferable to use them in sequence.

All UCSMP courses place continual emphasis on applications to the real world and to solving problems. UCSMP is designed to revitalize the traditional mathematics curriculum, improve student achievement, and increase the number of students who take mathematics beyond algebra and geometry.

The teacher's edition at each level provides the background, teaching suggestions and resources teachers need on a daily basis to implement the program. A Teacher's Resource File also provides support for teachers implementing the program. Support materials include: Lesson Masters, Teaching Aid Masters, Assessment Sourcebook, Technology Sourcebook, Answer Masters, Solution Manual, and a User's Handbook to implementing UCSMP. Also available are: Visual Aids, Activity Kit, Teacher's Resource File CD-ROM, Calculators, Study Skills Handbook, Geometry Template, *Wide World of Mathematics* Videotape/Videodisc/CD-ROM, Technology Sourcebook DataPak, Quiz and Test Writer Software, and other support materials.

## PROGRAM QUALITY

Reviewers found the six-year UCSMP materials stimulating and strong in all areas of mathematics. The program has clear, challenging learning goals consistent with NCTM standards. The content is well aligned with those goals and is carefully developed from problem situations to real-world experiences. The goals emphasize skill development and application.

The applications, illustrations, and examples are very engaging to students and teachers. Interactive software may be used to further increase student interest. The program makes a serious attempt to pre-

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## Professional Development Resources and Program Costs

Annual conferences at the University of Chicago and other regional meetings are offered by UCSMP and the publisher. Additionally, trained specialists are available to conduct school and district in-service meetings, and a number of newsletters are sent out each year to educators on the mailing list. (Details and costs of inservice meetings to be obtained from developer.)

For each course, curriculum and support materials can all be purchased separately. The student edition of the textbook ranges from \$42 for *Transition Mathematics*, *Algebra*, and *Geometry* to \$51.51 for *Functions*, *Statistics*, and

*Trigonometry* and *Precalculus and Discrete Mathematics*. Schools that purchase classroom sets of the UCSMP Student Editions receive the Teacher's Edition and Teacher's Resource File for each teacher at no charge. Optional materials vary by course and prices depend on the course. Scientific calculators are recommended for all courses. The *Explorations* CD-ROM (Macintosh/Windows) is recommended for *Functions*, *Statistics*, and *Trigonometry* and *Precalculus and Discrete Mathematics* and costs \$107.94 per course.

sent mathematical concepts by looking at patterns, proofs, problems, and examples. It thereby attempts to develop understanding in students and present mathematics in both inductive and deductive ways. The focus upon moving from specific examples, to conjectures, to generalizations, with constant review, is a strong component of the program. Every chapter makes connections with other disciplines and across the mathematical strands. During a lesson, students have an opportunity to gain depth of understanding of key conceptual ideas while explaining their thinking and understanding of a problem and its solution. The emphasis on students' communication of their reasoning makes higher order thinking an important attribute of the program. The use of long-term investigations is an excellent means for students to form a solid and deep understanding of mathematical concepts. UCSMP provides a variety of exercises for students. These are challenging and inviting to students including lower level students, students with language barriers, and accelerated students.

UCSMP includes a detailed, thorough, and carefully designed program of assessment. Teachers gain ongoing knowledge of student mastery of material from group activities and student assignments. The program recognizes the need for varied assessment devices so that students, regardless of learning style or ability, have ample opportunity to demonstrate the levels of learning they have attained. Students are also able to assess their own strengths and weaknesses as they complete assignments.

Both formal and informal methods of assessment are provided daily, and most follow the same format as the individual lessons and warm-ups. The alternate test forms and chapter projects and activities offer other types of assessment. Information is supplied to teachers on how to select assessment tools, how to score and evaluate them, and the use of a wide variety of assessment devices. Technology assessments are both within the lesson and included as separate activities. The close correlation between the learning goals and the individual assessment items provide unique strength to the assessment program.

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## *Usefulness to Others*

UCSMP is currently in use in diverse schools and districts across the country. By incorporating activities and alternative assessment ideas with the regular assignments and lessons, the program serves students with varying learning styles.

## *Educational Significance*

The program is based on the same principles that are the basis for the NCTM standards. The program is demanding and thorough, and the pedagogical and assessment approaches reflect the learning goals.

The program does an excellent job of presenting pertinent mathematics in contexts that are realistic. It also meets the needs of society and individuals by addressing the concerns that students learn more mathematics, that curriculum keep pace with the changes in mathematics and applications, and that students obtain the mathematics they need for employment and further schooling. The teacher's notes and supplemental information help improve their repertoire of instructional and assessment strategies.

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## PROGRAM EFFECTIVENESS AND SUCCESS

*The University of Chicago School Mathematics Project* (UCSMP) has been designated a **Promising** mathematics program.

The developers presented a number of evaluation studies as evidence of gains in student understanding of mathematics. For instance, for *Transition Mathematics*, *Algebra*, and *Advanced Algebra*, evaluations using a matched-group, post-facto, comparison design show that the UCSMP students had higher scores than comparison students on a number of subtests, although differences were not always statistically significant.

In 1985–86, data were collected in 20 matched comparison classes (7 in 7th grade, 10 in 8th grade, 3 in 9th grade) from the Chicago public schools, metropolitan Chicago area, and other states and towns. Students took the American Testronics High School Subjects Test: General Mathematics, a 40-question multiple choice test; the 60-item Orleans-Hanna algebra readiness test; and a 19-item geometry readiness test used in an earlier study at the University of Chicago. On virtually all tests, the *Transitions Mathematics* classes outscored the comparison classes significantly in geometry and algebra readiness and became more effective calculator users.

In 1987–88, a similar evaluation was conducted for the *Algebra* component of UCSMP with data collected from 11 matched pairs of classes in California, Colorado, Connecticut, Michigan, Ohio, South Carolina, and Wisconsin, representing urban, rural, and suburban areas. Students took the American Testronics High School Subjects Test: General Mathematics, a 40-question, multiple choice test (HSST), and two tests representing a wide range of Algebra and other mathematical concepts. The mean score of percent correct of test items for the UCSMP classes on the HSST was 1.2 percentage points higher than comparison classes. Similar results were found in a 1987–88 evaluation of the Advanced Algebra component; UCSMP students scored 15.6% higher than comparison students on a test using 50 multiple choice and four open-ended questions. Statistically significant positive differences favoring UCSMP students were found in three areas: reading, applications, and technology.

More recent studies confirm the trend toward positive effects for students on mathematics problem solving and applications. A 1992–1993 study indicated that, overall, students using UCSMP's 2nd edition of *Transition Mathematics* scored higher, to a statistically significant degree, than non-UCSMP students on the project-designed Algebra Posttest and project-designed Geometry Posttest. Additionally, students using UCSMP's 2nd edition of Algebra outperformed non-UCSMP students, to a statistically significant degree, on the project-designed Algebra Posttest and two other project-designed tests of problem solving and understanding. A 1993–1994 study found that all eight UCSMP 2nd edition *Geometry* classes in the study outperformed their non-UCSMP counterparts on the project-designed Geometry Posttest. However, the differences were not statistically significant.

## *Transition Mathematics*

Decimal Notation  
 Large and Small Numbers  
 Measurement  
 Uses of Variables  
 Patterns Leading to Addition  
 Problem-Solving Strategies  
 Patterns Leading to Subtraction  
 Displays  
 Patterns Leading to Multiplication  
 Multiplication and Other Operations  
 Patterns Leading to Division  
 Real Numbers, Area, and Volume  
 Coordinate Graphs and Equations

## *Algebra*

Uses of Variables  
 Multiplication in Algebra  
 Addition in Algebra  
 Subtraction in Algebra  
 Linear Sentences  
 Division in Algebra  
 Slopes and Lines  
 Exponents and Powers  
 Quadratic Equations and Square Roots  
 Polynomials  
 Linear Systems  
 Factoring  
 Functions

## *Geometry*

Points and Lines  
 The Language and Logic of Geometry  
 Angles and Lines  
 Objectives  
 Proofs Using Congruence  
 Polygons and Symmetry  
 Triangle Congruence  
 Perimeters and Areas  
 Three-Dimensional Figures  
 Surface Areas and Volumes  
 Indirect and Coordinate Proofs  
 Similarity  
 Similar Triangle and Trigonometry  
 Further Work with Circles

## *Advanced Algebra*

Functions  
 Variations and Graphs  
 Linear Functions  
 Matrices  
 Systems  
 Quadratic Functions  
 Powers  
 Inverses and Radicals  
 Exponential and Logarithmic Functions  
 Trigonometry  
 Polynomials  
 Quadratic Relations  
 Series and Combinations

## *Functions, Statistics, and Trigonometry*

Exploring Data  
 Functions and Models  
 Transformations of Graphs and Data  
 Circular Functions  
 Trigonometric Functions  
 Root, Power, and Logarithm Functions  
 Probability and Simulation  
 Sequences, Series, and Combinations  
 Polynomial Functions  
 Binomial and Normal Distributions  
 Matrices and Trigonometry  
 Quadratic Relations  
 Further Work with Trigonometry

## *Precalculus and Discrete Mathematics*

Mathematical Logic and Reasoning  
 Analyzing Functions  
 Functions, Equations, and Inequalities  
 Integers and Polynomials  
 Rational Numbers and Rational Functions  
 Trigonometric Identities and Equations  
 Recursion and Mathematical Induction  
 Polar Coordinates and Complex Numbers  
 The Derivative in Calculus  
 Combinatorics  
 Graphs and Circuits  
 Vectors  
 The Integral in Calculus





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